Adiabatic Theory of Electron Oscillations and its Application to SIS100/SIS200.

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1. Main features of the model.

- Account only the ionization electrons.
- One-dimensional (vertical) electron oscillations.
- An ion and electron beams have round cross-sections and Gaussian distribution in transverse direction.
- Electron stability inside the gaps between bunches appears because of ion leakage from the bunch.
- Bunch gaps result in creation of „survival layer“ where electron oscillations are stable during long time.
- Electrons are heated by Coulomb collisions with the circulating ions.
- Electron concentration in survival layer is defined by balance between electron creation due to ionization process and electron loss due to Coulomb heating.

2. Machine characteristics.

In GSI new ion synchrotron complex is now under design.

It includes two synchrotrons: SIS100 and SIS200.

The most dangerous periods from point of view of electron accumulation:

In SIS100= injection process;

In SIS200 = “stretcher” mode.

Parameters of both machines in these regimes are given in Table 1.

Table 1. Parameters of SIS100 and SIS200.

Circumference (m)	1080	1080
Energy (Mev/u)	100	1000
Process time (s)	1	1
Number of bunches	4	None (1)
Kind of ions	$^{238}U^{+28}$	$^{238}U^{+28}$
Number of ions in each bunch N_b	2.5×10^{11}	10^{12}
Bunch length L_b (m)	216	(864)
Rms vertical bunch size a_v (m)	0.015	0.01
Rms horizontal bunch size a_h (m)	0.015	0.01
Vacuum chamber radius	0.05	0.05
Pressure (10^{-10} mbar, without beam)	0.05	0.1

5 possible schemes of bunch location during the injection in SIS100:

1) Scheme number 1: only one bunch in the ring.

2) Scheme number 2: two bunches in opposite separatrices.

3) Scheme number 3: two bunches in neubour separatrices.

4) Scheme number 4: one bunch is absent.

5) Scheme number 5: all bunches are present.

In stretcher mode (SIS200) it is used the coasting beam.

If the electron concentration is too high in principle it is possible to use one bunch regime.

2. Transverse electron oscillations and adiabatic invariant.

Dimensionless equation of transverse electron oscillations:

$$y'' + (2pQ_0)^2 F(t)\, y\Phi(x, y) = 0$$

$y = Y / a_e, x = X / a_e$,

Y, X =vertical and horizontal electron deviations,

$t = t/T$, where t =time, T =period of the ion line density variation.

Q_0 = „average electron betatron tune“, equaled to number of betatron oscillations on the bunch length for uniform ion density,

$$\Phi(x, y) = \frac{1 - \exp[-(x^2 + y^2)]}{x^2 + y^2}$$ for round Gaussian beam,

$F(t) = \dfrac{Z_i\, l_i(t) - l_e}{Z_i \langle l_i(t) \rangle}$, where $l_i(t)$ =ion longitudinal density inside the bunch,

$\langle l_i(t) \rangle$ = mean value of $l_i(t)$, l_e = electron longitudinal density (uniform).

„Average betatron tune“ $Q_0 = \sqrt{\dfrac{N_b r_e Z_i R}{p b^2 a^2 h}}$, where r_e -classical electron radius, N_b-number of ions inside the bunch, *ß* -ion relativistic parameter, Z_i=charge ion number, R=ring radius, h=number of bunches.

Four models of $F(t)$ (with uniform density in the gap):

1. “Square” model with uniform density in the bunch.
2. “Elliptical model” with elliptical density in the bunch.
3. “Cosine model” with flat top of the bunch and cosine law in the bunch edge.
4. Smooth “cosine model” with cosine density in the bunch.

The first model has breaks in function and its derivative, the second one - only in derivative, the third and fourth functions are continious with derivatives.

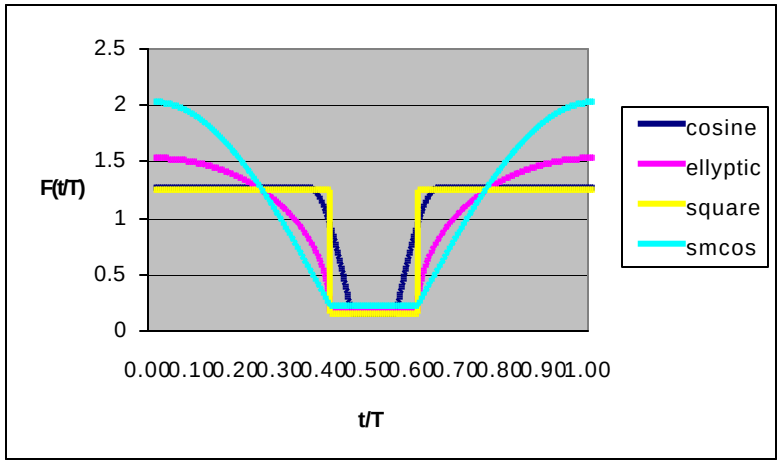


Fig. 1. Different longitudinal distributions used in calculations.

Adiabatic invariant $I(y_{\max},t)=4\int_0^{y_{\max}} y'dy$,

Maximal value of action corresponds to the gap center (t =0.5) and $y_{\max}=b$.

In another points of the bunch $y_{\max}$ is defined by the equation:

$$I(y_{\max},t)=I(b,0.5)$$

Example of dependence $y_{\max}$ on t for different values of parameters c,h;

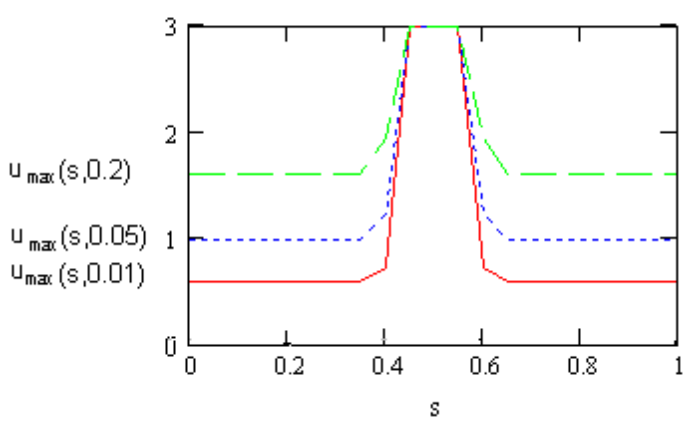


Fig.2. Dependence of normalized electron beam size $u=y_{\max}(s,c)$ on t for different values of parameter c (h =0).

„Gap density parameter“ c =relation of ion density in the gap to ion density in the center of bunch, „neutralization degree“ $h = N_e / Z_i N_i$,

N_e = number of electrons in the ring, N_i = number of ions in the ring.

Adiabaticity criterion (Landau and Lifshits, „Mechanics“)

$$K_{ad} = \frac{d\Omega(t)}{dt} T / \Omega(t) = \frac{dF(t)}{dt} T / F(t) << 1$$

The adibaticity criterion depends on form of longitudinal disitribution, as well as from variables c, h, t ; it reaches maximal value near the bunch edge.

Example of trajectory

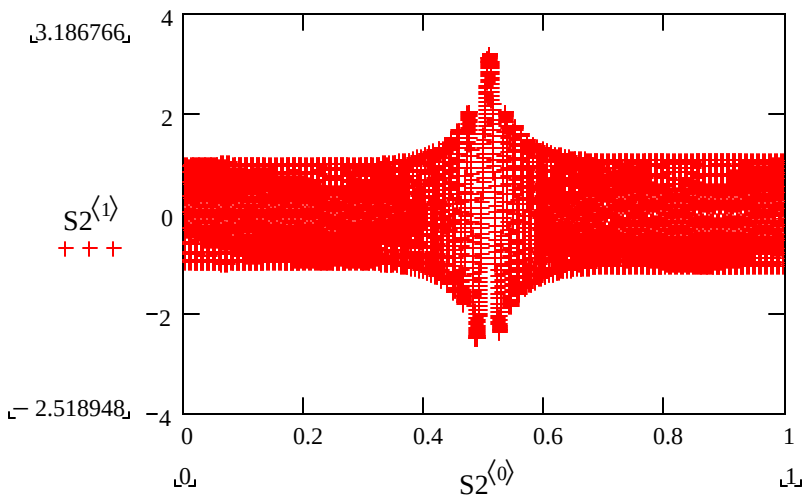


Fig. 3. Trajectory for c =0.01. Maximal deviation is equal to 3.19 (in accordance with adiabatic theory 3.16).

3. Linear oscillations.

Adiabatic solution in linear case:

$$y = aj(t) + a^* j(t)^*, j(t) = \frac{\exp[i2pQ_0 \int_0^t \sqrt{F(t_1)} dt_1}{\sqrt{2pQ_0 \sqrt{F(t)}}}$$

Adiabatic invariant $I = 4|a|^2$. Adiabaticity perturbations result in amplitude perturbations! Equation for complex amplitude :

$$a' = -\frac{i}{2}\{a j(t)[-\frac{\Omega''}{2\Omega} + \frac{3(\Omega')^2}{4\Omega^2}] + C.C.\} j^*(t)$$

The resonance strength can be calculate using standard matrix procedure.

Eugen values of transfer matrix of the period M_T are:

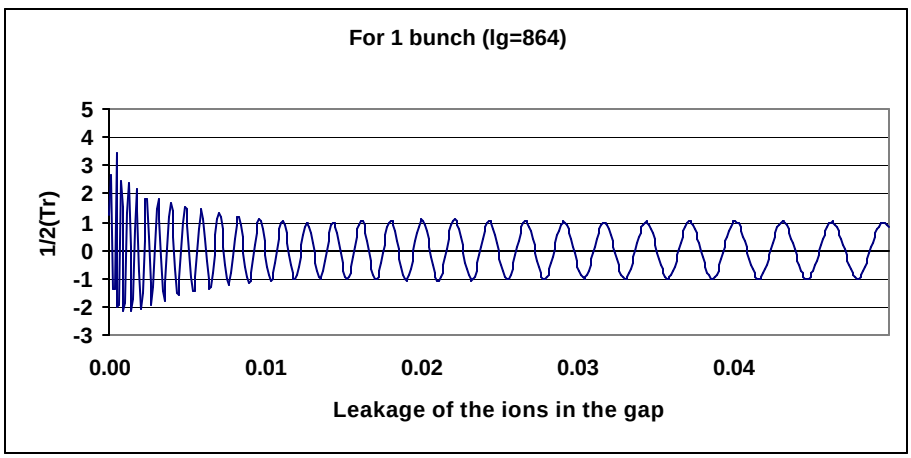
$$l_{1,2} = Tr(M_T)/2 \pm \sqrt{[Tr(M_T)/2]^2 - 1}$$

If $|Tr(M_T)|$<2, eugen values are imaginary and the motion is stable.

At the opposite case a motion is unstable. Resonance strength

$$g = \ln[|Tr(M_T)|/2 + \sqrt{[Tr(M_T)/2]^2 - 1}]$$

At Fig.4 we see „classical“ picture: dependence $Tr(M_T)/2$ on leakage factor.



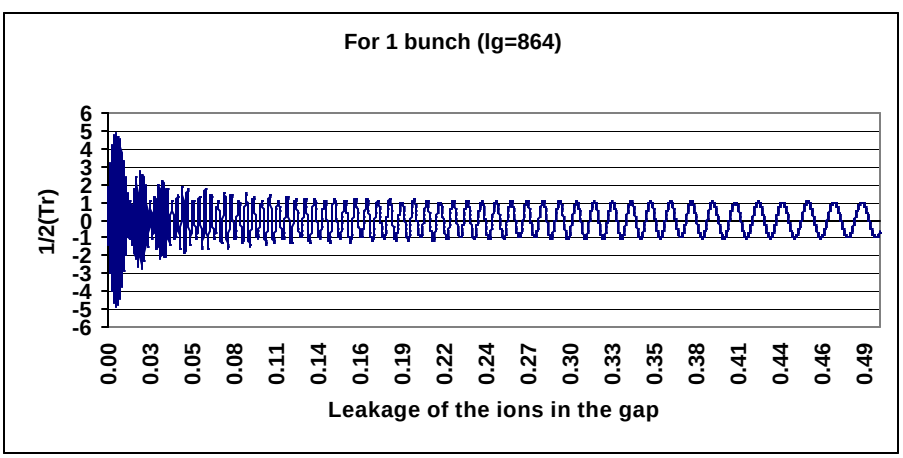
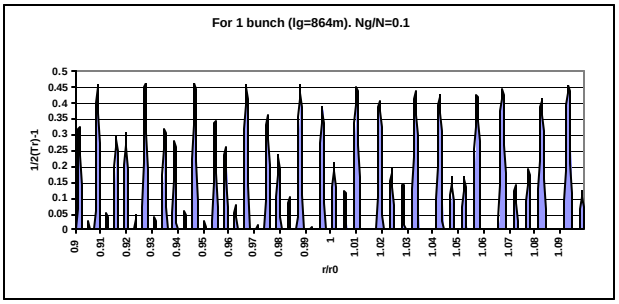


Fig. 4. Dependence of K_{foc}^{1} on m (top picture-smooth cosine model, bottom picture-elliptical model, 1 bunch).



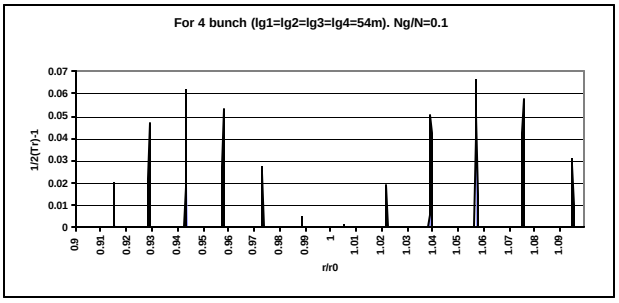
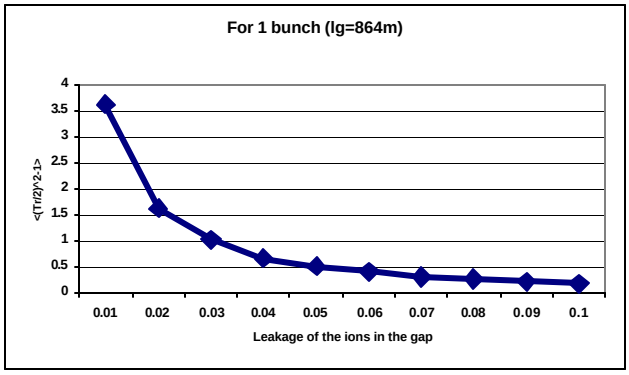


Fig. 5. Dependence of K_{foc}^{2} on the beam size for elliptical model, m =0.1 (top picture-1 bunch, bottom picture -4 bunches).



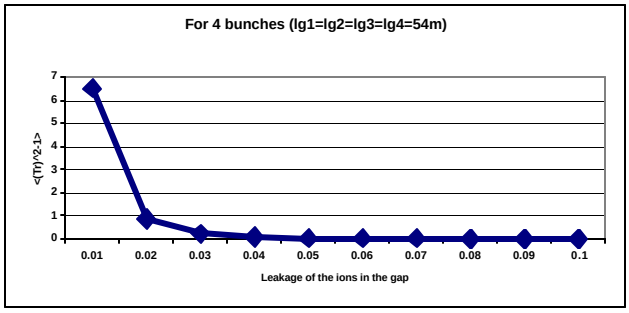


Fig. 6. Dependence of K_{foc}^{3} on m for elliptical model (1 bunch- top picture, 4 bunches- bottom picture).

The heating rage due to resonances is defined by $K_{foc}{}^3 = \langle (Tr/2)^2 - 1 \rangle$:

$$\left\langle \frac{dI}{dt} \right\rangle \approx \frac{\langle I \rangle}{8T} K_{foc}{}^3$$

4. Non-linear oscillations.

Y =maximal amplutude of surviving electrons,

Ni0= bunch intensity and neutralization h =0 (“cosine model”).

If $N \to \infty$, $Y \to Y(c)$ (“physical aperture”).

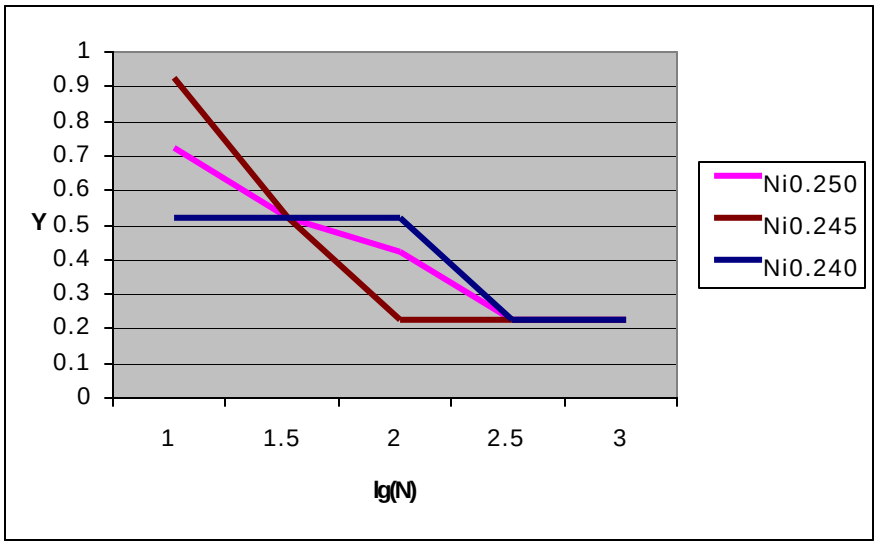


Fig.7. SIS100, 4 bunches, dependence of $Y = y_{ap}(c, h, N)$ on N=t/T, (T = time duration of one bunch), c =0.1, h =0.

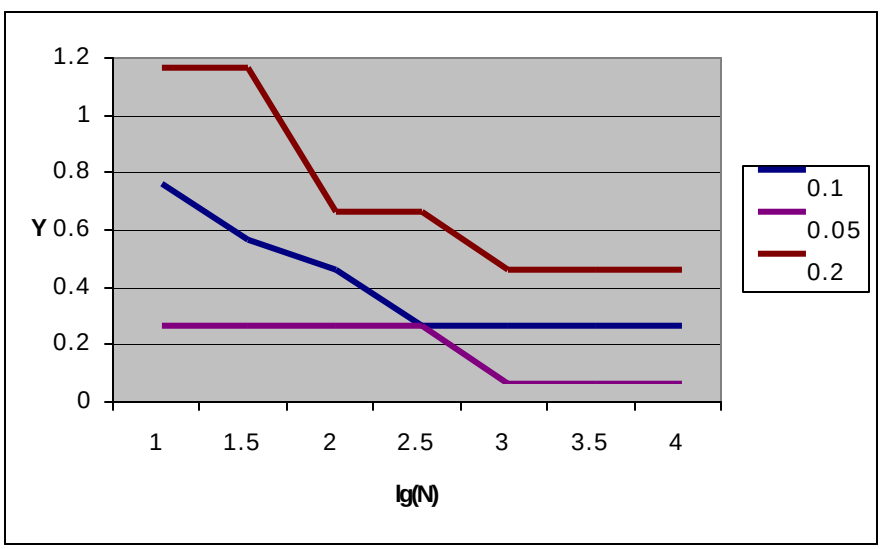


Fig.8. SIS100, 4 bunches, dependence of $Y = y_{ap}(c,h,N)$ on $N=t/T$, (T = time duration of one bunch) for different values of c ($Ni0 = 0.25 \cdot 10^{12}$), h =0.

The similar calculations are made for SIS200, 1 bunch, h =0.

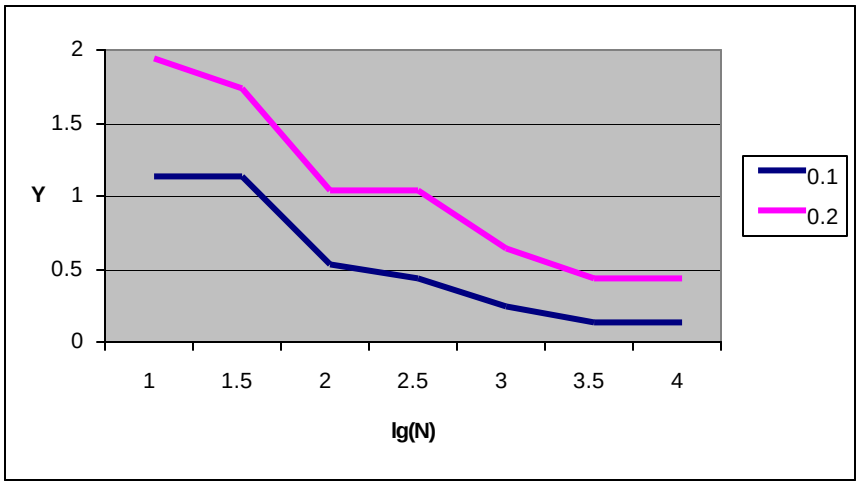


Fig.9. SIS200, dependence $Y = y_{ap}(c,h,N)$ on $N=t/T$, (T = time duration of one bunch) for different values of c , ($Ni0 = 10^{12}$), h =0.

At the following text we assumed that electrons have long life only if

$$Y < Y_{ap}(c)$$

5. Equilibrium neutralization degree.

Coasting beam (Zenkevich, meeting on Space Charge Dominated beams, Japan).

Electron creation due to ionization.

Electron heating due to Coulomb collisions with the ions.

Electron is lost when energy is more than maximal energy of the potential well.

Main Equation:

$$\frac{dh}{dt} = \frac{1}{T_{neutr}} - \frac{h}{(1-h)T_{life}^{\ 0}}$$

T_{neutr}=time of creation Z_i electrons by each ion, $T_{life}^{\ 0}$ –electron lifetime for h =0.

Equilibrium neutralization degree

$$h^{eq} = \frac{h_0^{eq}}{1+h_0^{eq}}, h_0^{eq} = K_0 a^2 P \langle \Phi_m(b) \rangle$$

where a in cm, P in 10^{-10} mbar, and constant $K_0 = \frac{10^{-13} N_{Losh} K}{2 r_e L_{coul}} = \frac{0.0992}{L_{coul}}$.

Function $\Phi_m(b)$ appears from ionization cross-section (Baconnier et al):

$$\Phi_m(b) = C_m + M_m^2 (\ln \frac{b^2}{1-b^2} - b^2)$$

parameters C_m and K_m depend on kind of gas, sign $\langle \ \rangle$ means averaging on the components.

h_0^{eq} is independent on the ion charge Z_i and very weak dependent on b !

Bunched beam (scaling).

Main idea: development of survival level.

Equation:

$$\frac{dh}{dt} = \frac{1}{T_{neutr}(c,h,t)} - \frac{h}{T_{life}(c,h,t)}$$

Solution can be found for small neutralization and $t > T_{life}$. Then:

neutralization time

$$T_{neutr} \approx T_{neutr}^0 / Y_{ap}(c)$$

electron lifetime in the layer

$$T_{life} \approx (T_{life})_{coast} Y_{ap}(c)^2$$

equilibrium neutralization degree

$$h_0(c) \approx h_0^{eq} Y_{ap}(c)^3$$

In physical variables: $Y_{ap}(c)$ =relation of maximal oscillation amplitude of long lived electrons at the bunch center to rms beam size a, depending on leakage factor.

6. Neutralization degrees in SIS100 and SIS200.

In SIS200 for coasting beam the neutralization degree h =0.6-0.8 % (nominal vacuum). With beam h can be more at order of magnitude!

It can be diminished by beam bunching. Let the beam is bunched during the stretching in one bunch (bunch length=80% from the circumference).

Table 2. Neutralization parameters for SIS100-SIS200: kind of ions $^{+28}U^{238}$, in SIS100 P= 5*10^{-12} mbar, in SIS200 P= 10*10^{-12} mbar, gas composition coincides with mesured gas composition in SIS-18 (H_2=65%, $0/H_20$=17%, CO/N_2=8%, Ar=4%, Cl=4%, CO_2=1%).

Machine	SIS100 c =0.1	SIS100 c =0.2	SIS200 c =0.1	SIS200 c =0.2
$\langle s_{ion} \rangle$ (10^{-16} cm^2)	10.3	10.3	7.25	0.05
t_{neutr}^{0} (s)	15.6	15.6	13.1	13.1
t_{life}^{0} (s)	0.102	0.102	0.092	0.092
$h_0^{eq} = t_{life}^{0} / t_{neutr}^{0}$	6.5*10^{-3}	6.5*10^{-3}	7.0*10^{-3}	7.0*10^{-3}
$Y(c)$	0.2	0.4	0.1	0.4
$t_{life}(c)$	0.0204	0.0408	0.0092	0.368
$h_0(c) = h_0^{eq} Y(c)^3$	5.2*10^{-5}	4.2*10^{-4}	7.0*10^{-6}	4.5*10^{-4}

We see that the calculated neutralization degree are very small!

Possible drawbacks:

1) sharp vacuum decrease due to beam;

2) other sources of electrons (ion-electron secondary emission, SEM).

Future plans: more detailed code using Monte Carlo Method.