# *3D Particle in cell program for Electron Cloud*

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# Motivation and contents

## Motivation

- To study the mechanism of electron motion in different fields
- To study the multi-pacting effects in KEKB LER

## Contents

### Topic one

Multipacing effects in various fields

### Topic two

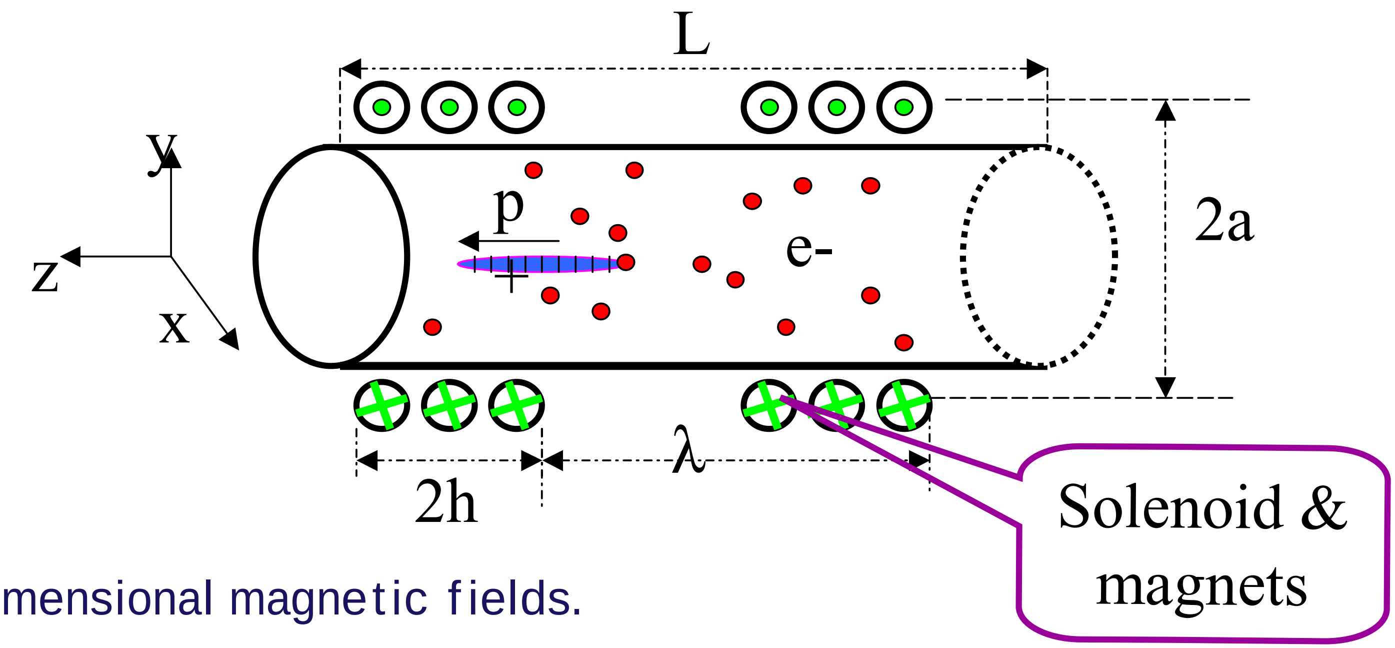
Photoelectron Trapping in Quadrupole and sextupole magnets

### Summary

# Program---picec3

## Program model

- PIC methods




## Magnetic field

- General 3-dimensional magnetic fields.
- Fields can be import from other program

## Beam potential

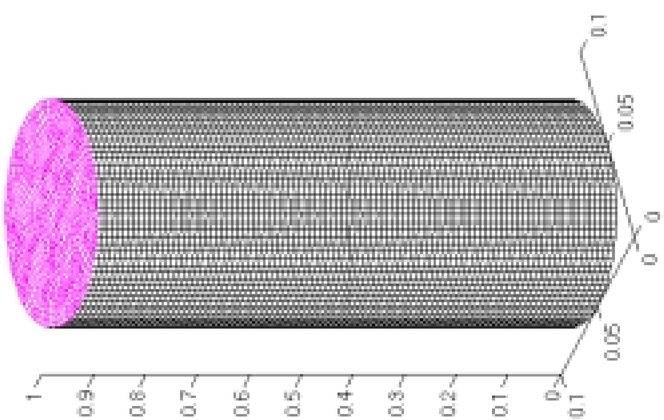
- Gaussian bunch in round chamber

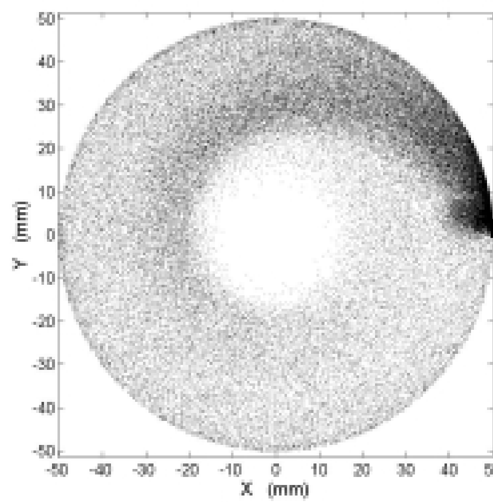
$$U = \frac{eN}{4\pi\varepsilon_0} \int_0^\infty \frac{\exp[-\frac{x^2}{2\sigma_x^2+t} - \frac{y^2}{2\sigma_y^2+t})]}{(t+2\sigma_x^2)^{1/2}(t+2\sigma_y^2)^{1/2}} dt$$

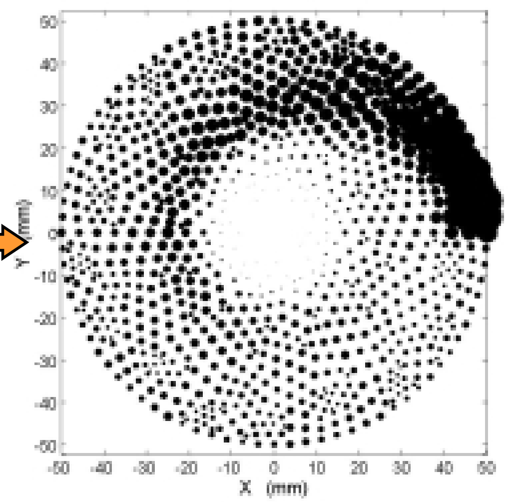
# Program- - -picec3

## *Space charge potential of e-cloud*

- 3 dimensional irregular mesh to better represent the general chamber geometry, or to handle accuracy with high order elements.
- Powerful tool for solving poisson problems (beam potential, magnetic field,etc.).






***Mesh of chamber***

***Real charge distribution***

***Meshed Charge distribution***

# Program---picec3

## *Integration of the particle motion equation*

- Exact formula for uniform fields(drift region, dipole magnet,uniform solenoid)
- Numerical integral for general magnetic field cases.
- Difficult to simulate e-cloud motion efficiently.
  - Short bunch
  - Strong magnetic field
  - 3Dimensional irregular mesh for space charge.
  - Larger number of particles.

# Simulation Parameters (KEKB LER)

- $C=3016.26$ m
- $E_0=3.5$ GeV
- $\sigma_l=4$ mm
- $F_{RF}=508.89$ MHz
- $h=5120$
- $\beta_x=\beta_y=10$ m
- $\varepsilon_x=1.8\times10^{-8}$ m
- $\varepsilon_y=3.6\times10^{-10}$ m
- $r_{pipe}=0.05$ m
- $N=3.3\times10^{10}$

- $Y'=0.1$
- $R=0.3$
- $e_0=5.0$ eV
- $\sigma_{e1}=5.0$ eV
- $\sigma_{e2}=5.0$eV

$\varepsilon_{\max}=250eV$

$\delta_{\max}=1.5$

- Bunch spacing=8ns

# Secondary emission

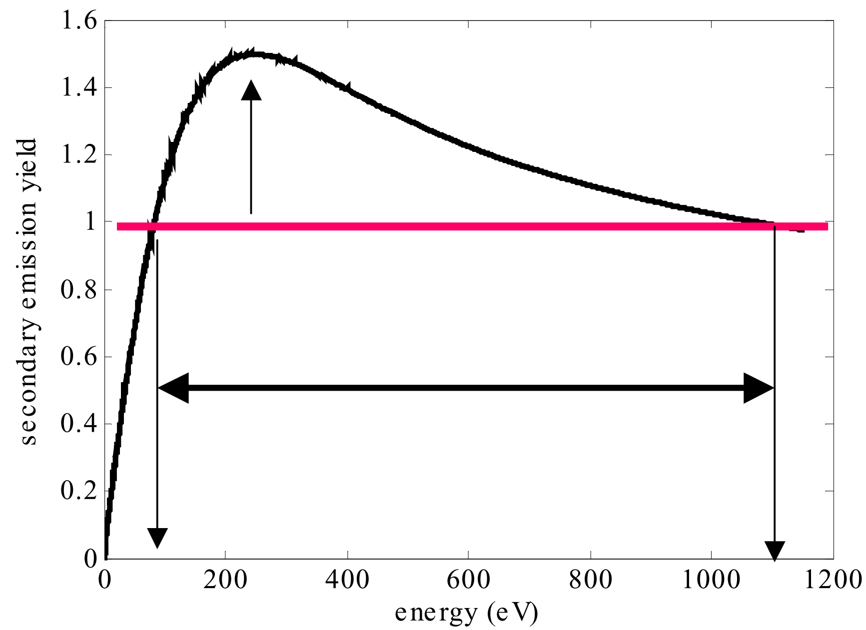
## Secondary emission yield

$$Y_2(\varepsilon,\theta) = \delta_{\max} 1.11 x^{-0.35} (1 - e^{-2.3x^{1.35}}) e^{0.5(1-\cos\theta)}$$

$$x = \varepsilon_p (1 + 0.7(1 - \cos\theta)) / \varepsilon_{\max}$$

$\varepsilon_{\max} = 250eV$ $\delta_{\max} = 1.5$

The charge of marc-particle should change according to the secondary emission yield

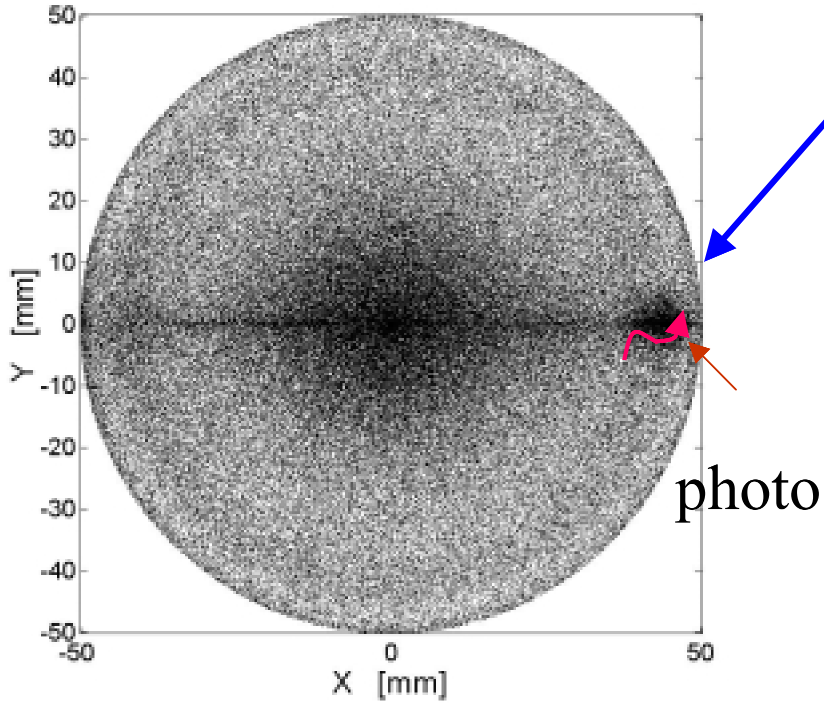



## Secondary electron energy

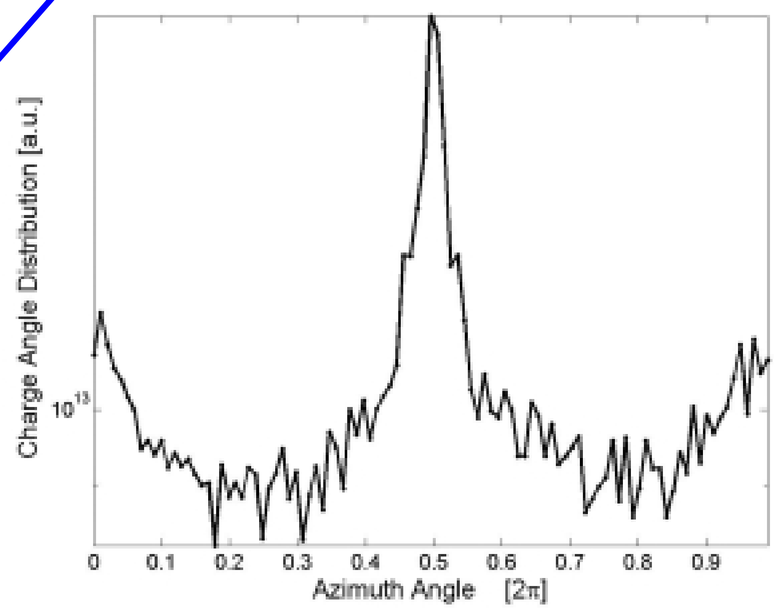
$$\delta_E(\varepsilon) = e_{20} \exp\left( -\frac{\varepsilon}{2e_{20}^2} \right)$$

$$e_{20} = 5eV$$

# Field free region



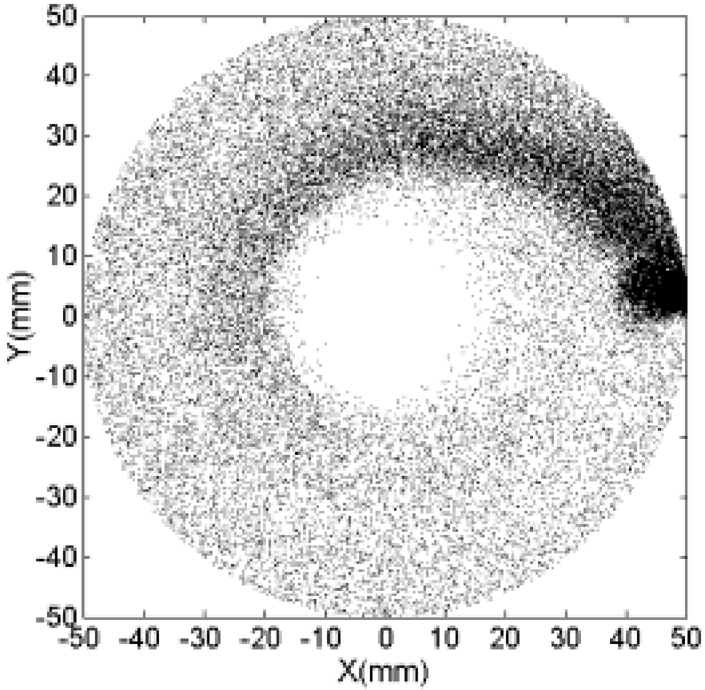

*Live Cloud Distribution in Transverse Plane*



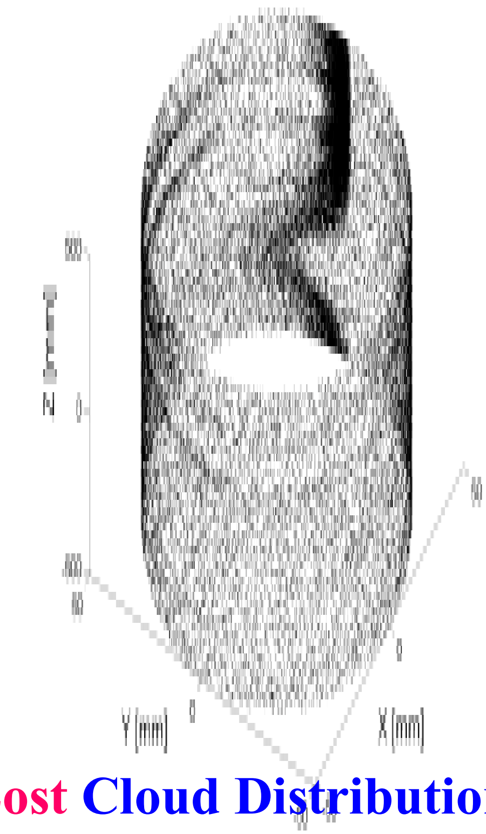

*Charge Azimuth Distribution of the Lost Cloud*

***Multipacting+large central density+heating***

# Solenoid



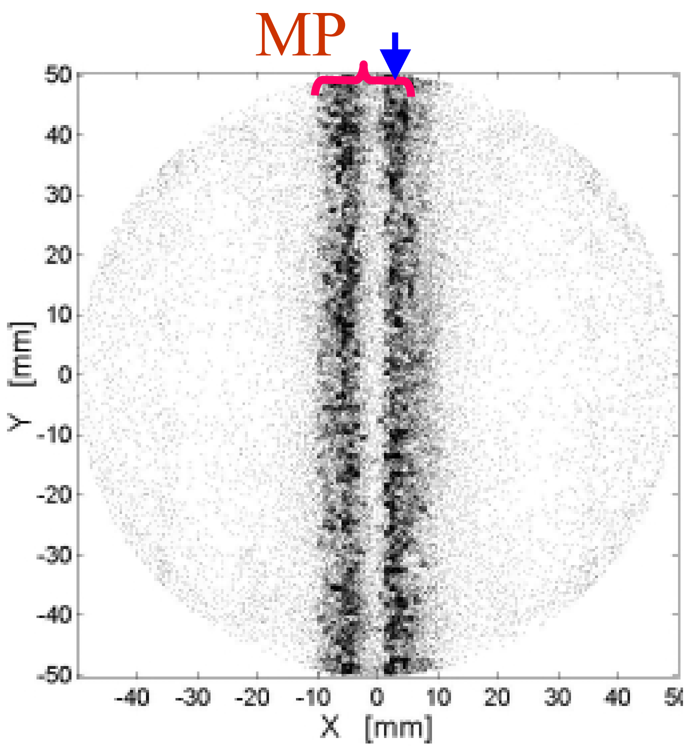

**Live Cloud Distribution**



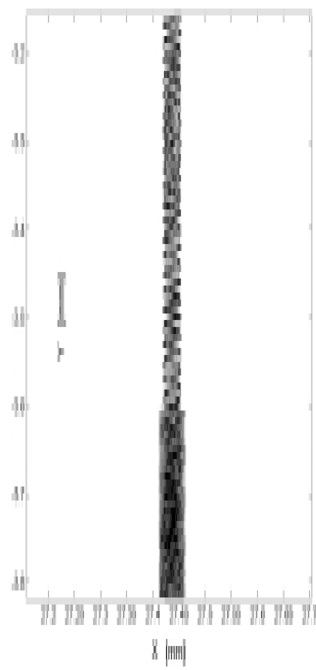
**Lost Cloud Distribution around the chamber wall**

***No Multipacting+zero central density+low level heating***

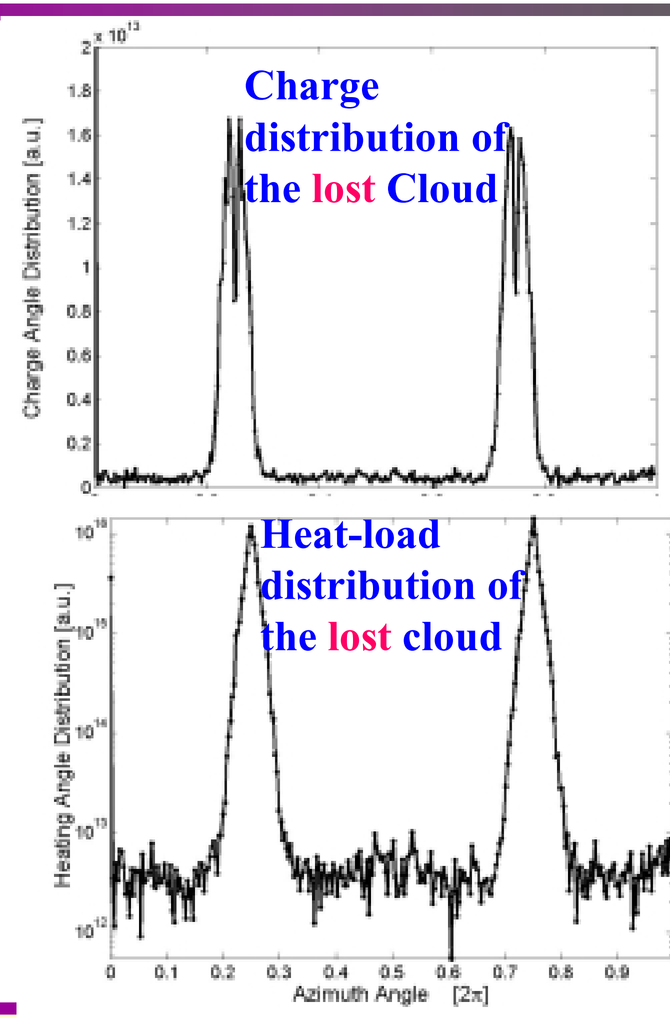
# *Normal Dipole*




*Live Cloud Distribution*



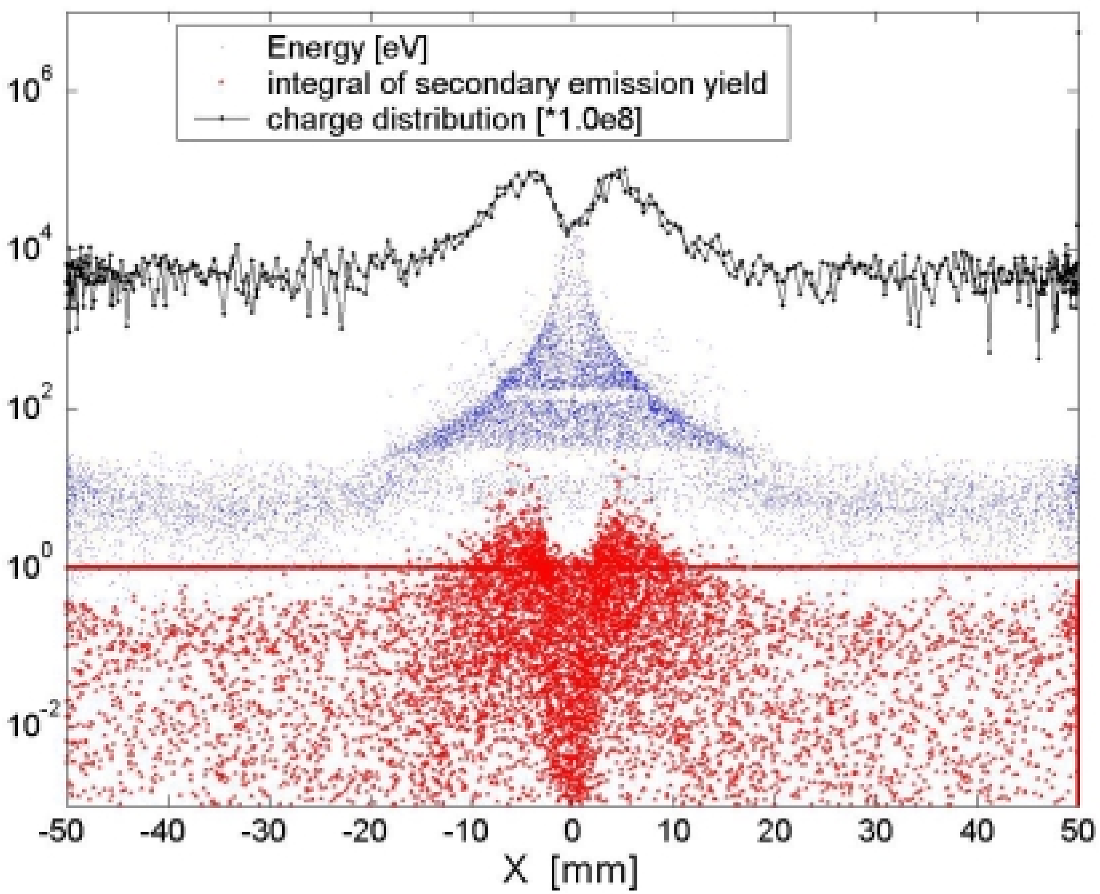
*Transverse orbit*

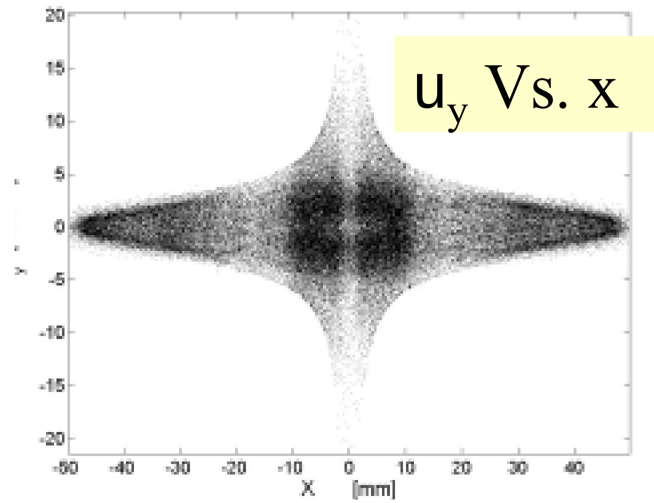


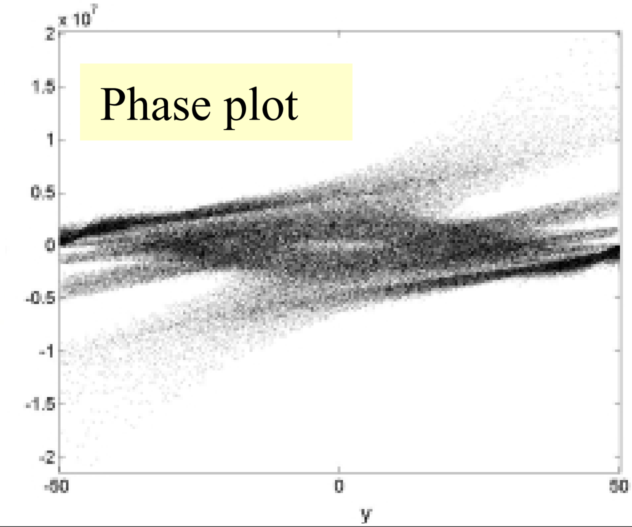

***Low central density +local multipacting+local heating***

## ***Normal Dipole***

***(Contd.)***

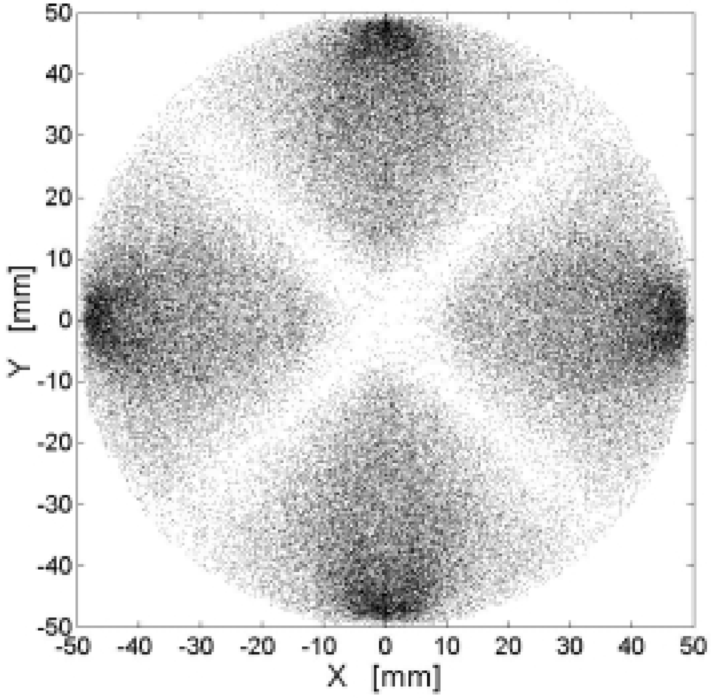


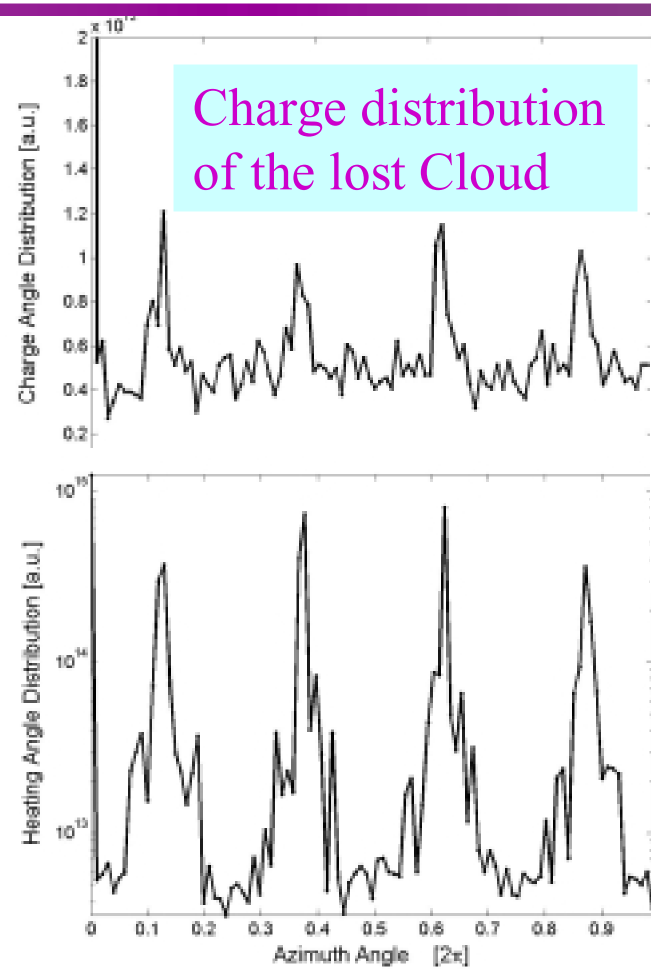



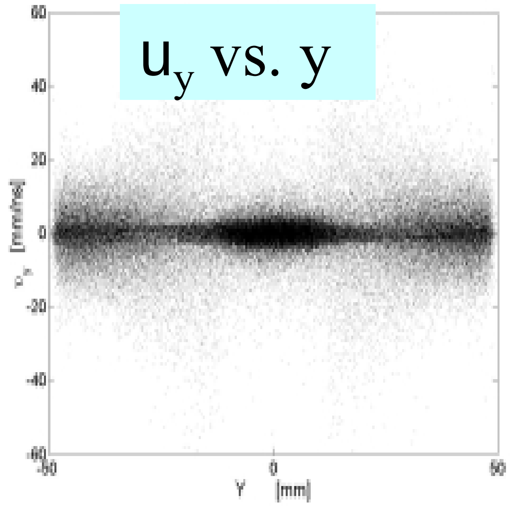

***Multi-pacting mechanism***

## *Normal Quadrupole*




***Live Cloud Distribution***

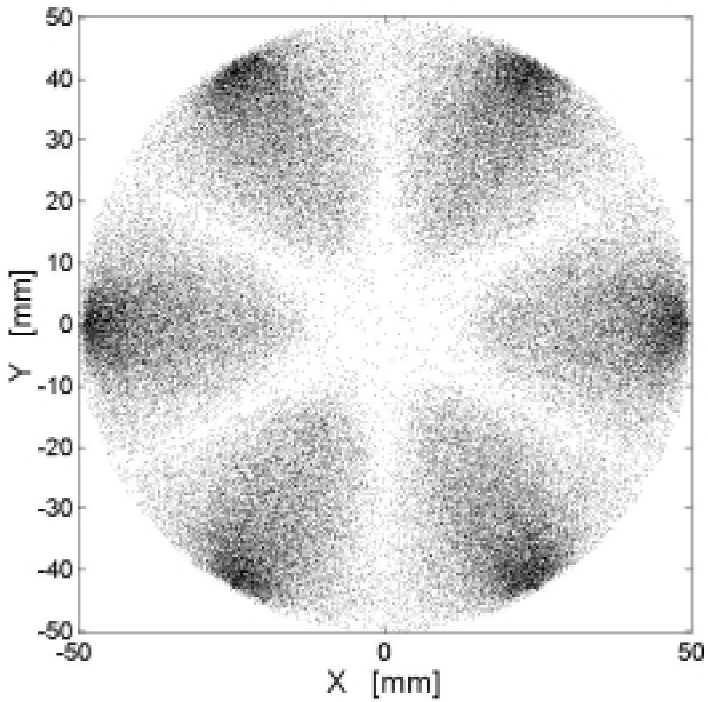


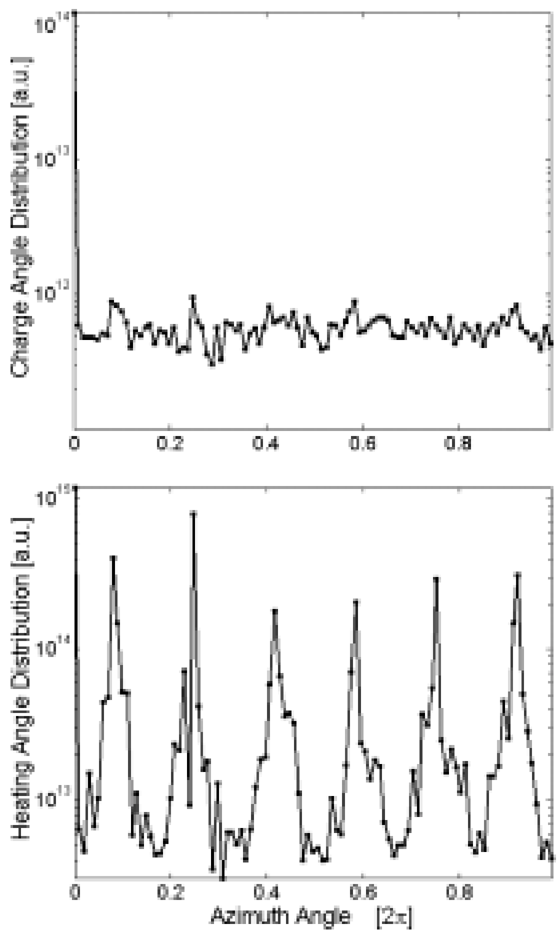

Heat-load distribution of the lost cloud

***Weak multipacting+low central density+weak heating+trapping***

# Normal Sextupole




*Live Cloud Distribution*

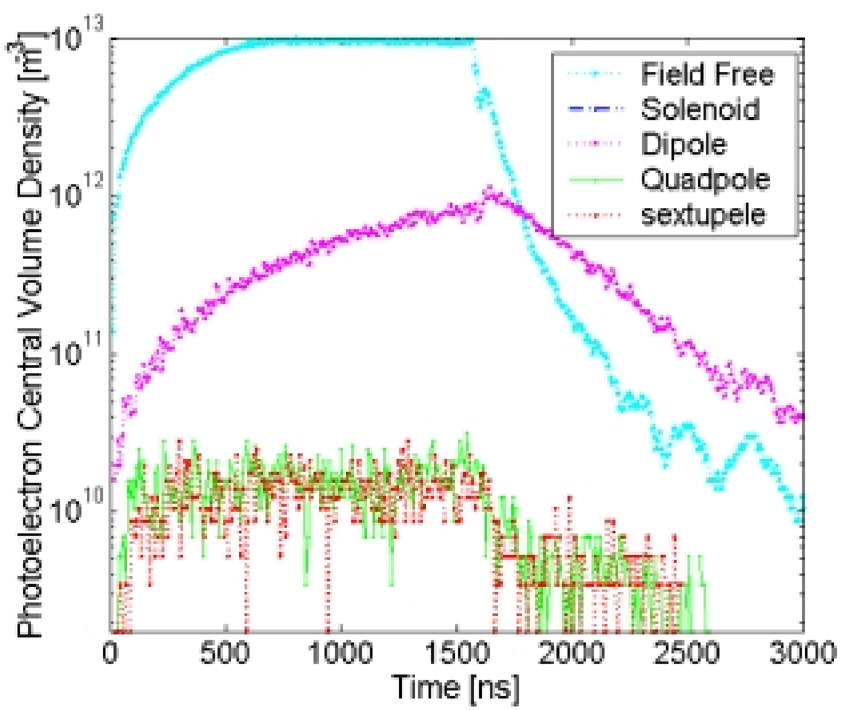



Charge distribution of the lost Cloud

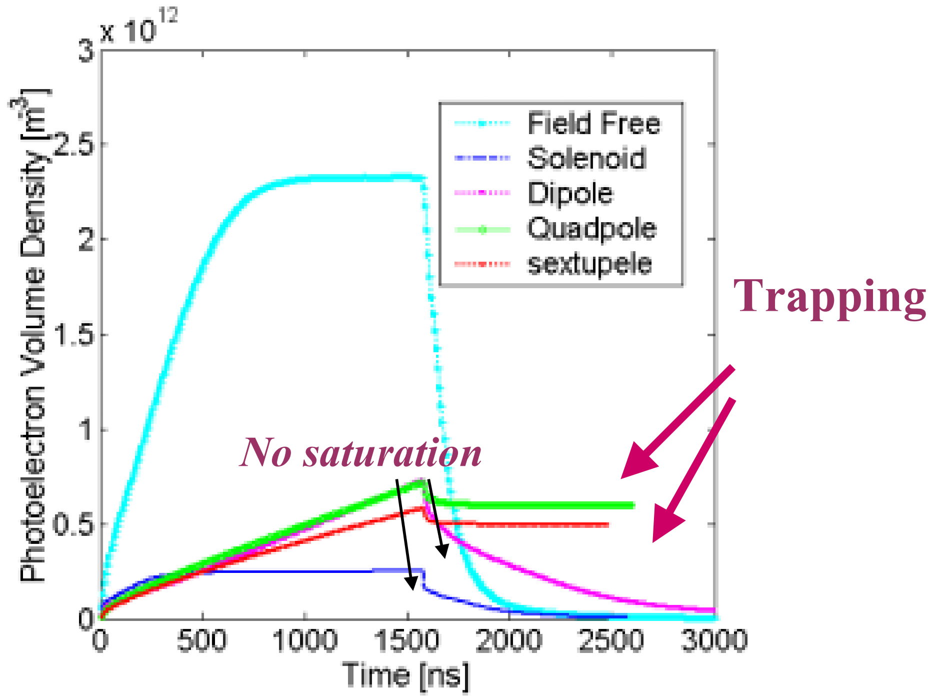
Heat-load distribution of the lost cloud

***Weak multipacting+low central density+weak heating+trapping***

# Cloud Density in Different Fields



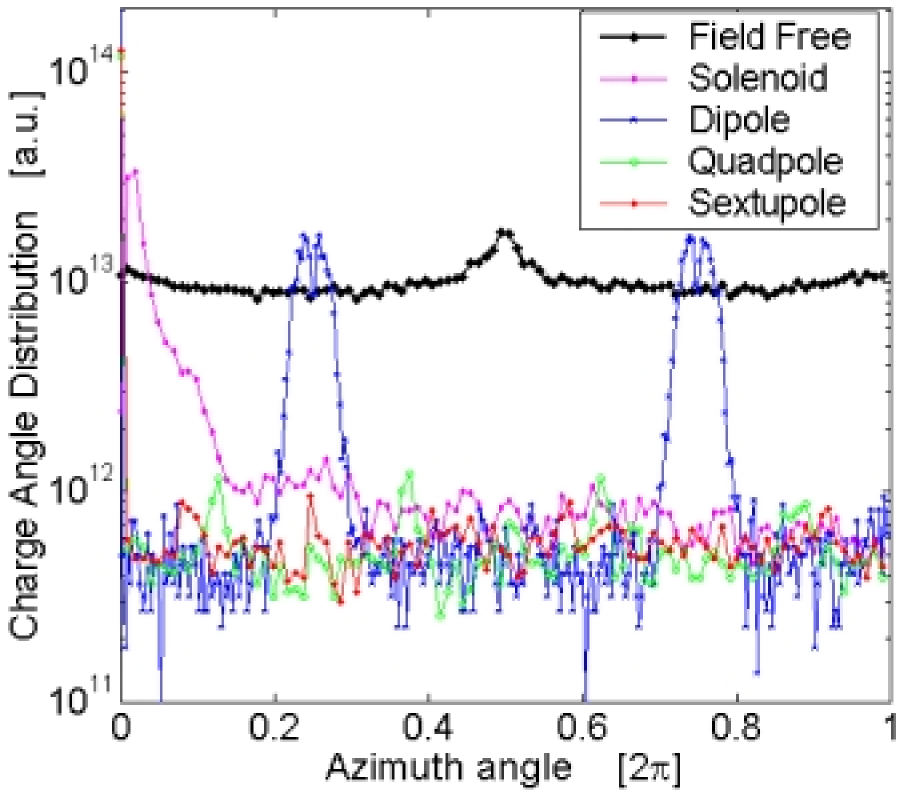

*Central density*



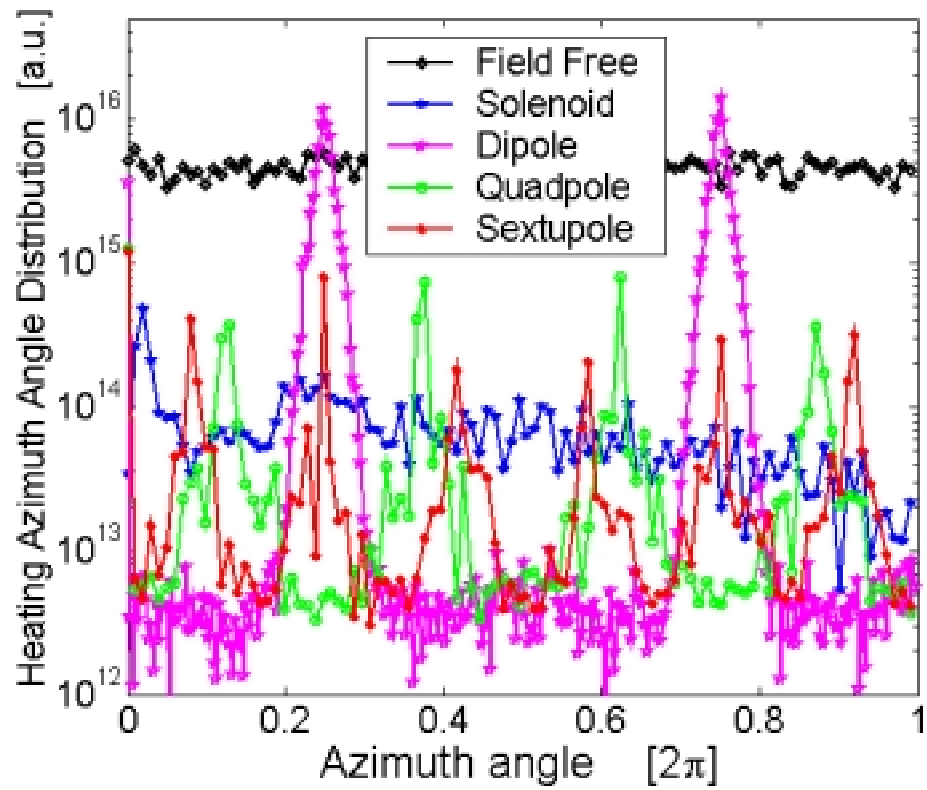

*Average density*

***Electron volume density as a function of time for a train with 200 bunches spaced by 7.86 ns and followed by a gap***

# *Electrons collosion with wall*




Charge Azimuth Distribution of the Lost Cloud




Heating Azimuth Distribution of the Lost Cloud

**Novel photoelectron trapping in quadrupole and sextupole magnets**

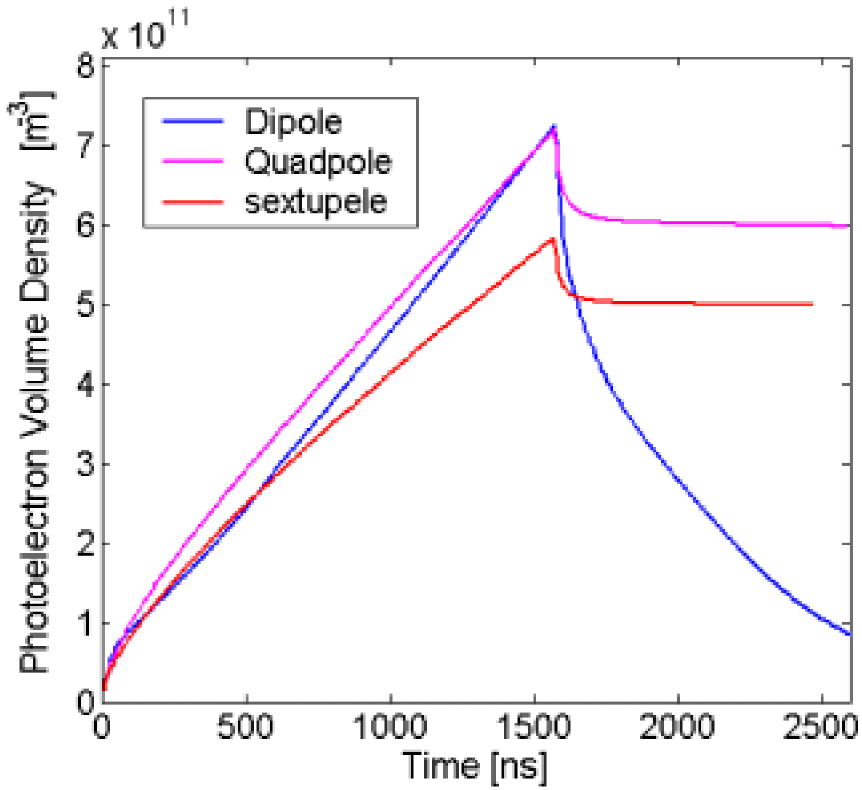
→ **Mirror Field Trapping+beam effects**

First reported the ***phenomenon*** in two-stream seminar, KEK, ***2001,Sep.***

First explained the ***mechanism*** in Seminar of KEKB Instability Working Group, ***２００１,Nov. 20***

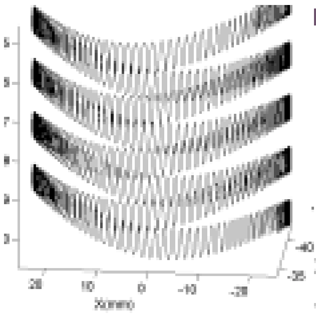
## Trapping phenomenon

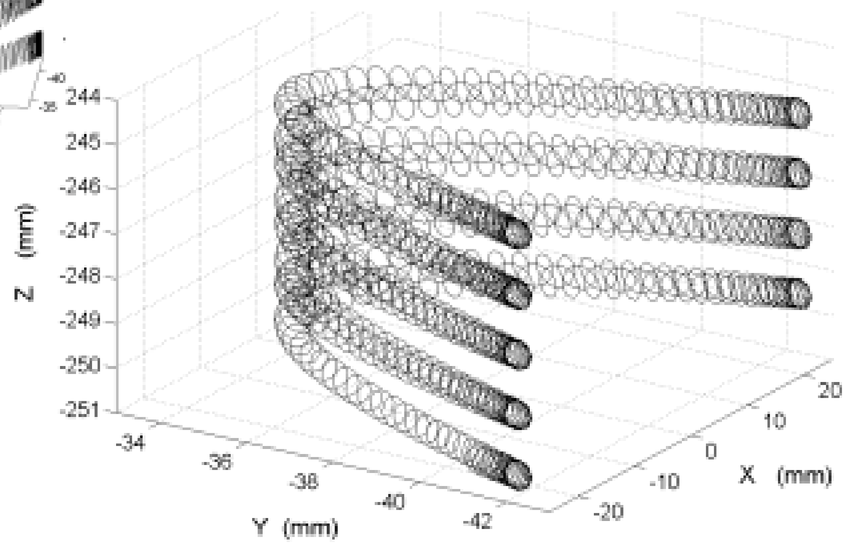
- It happens in quadrupole and sextupole magnets
- The photoelectron can be trapped in quadrupole and sextupole magnets for very long time until it longitudinally drift out of the magnets.($v_z$~0.004 mm/ns)
- The trapping phenomenon is strongly beam-dependent. There is no such kind of trapping when the positron beam force is not included

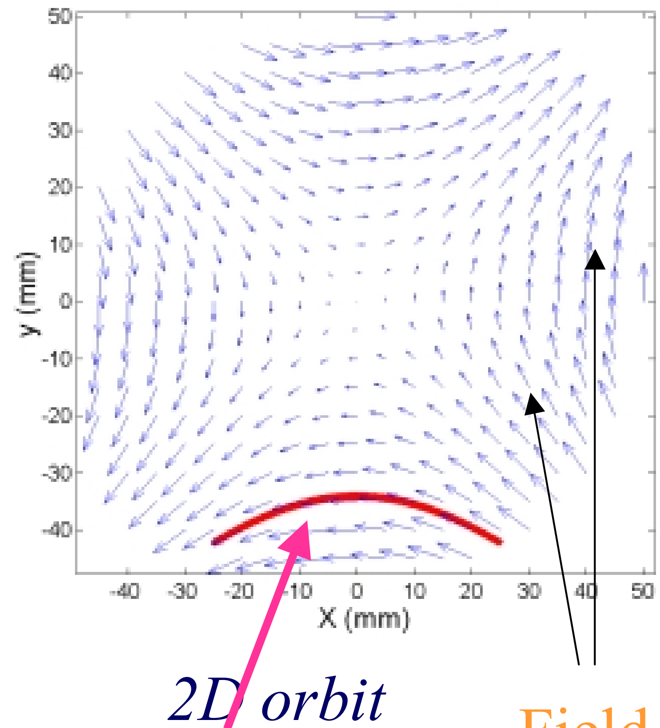



***Average cloud density evolution in different magnetic fields***

# Trapping phenomenon---in quadrupole magnet






*3D orbit*

*2D orbit*

Field lines

***Orbit of a trapped photoelectron in normal quadrupole magnet during the train gap*** (field gradient=0.5T/m)

- The motion of the electron in magnetic field can be regarded as the superposition of the gyration motion around the guide center and the motion of the guide center.

## *Adiabatic invariation*

***For periodic motion***

$$J = \oint p\,dq = constant$$

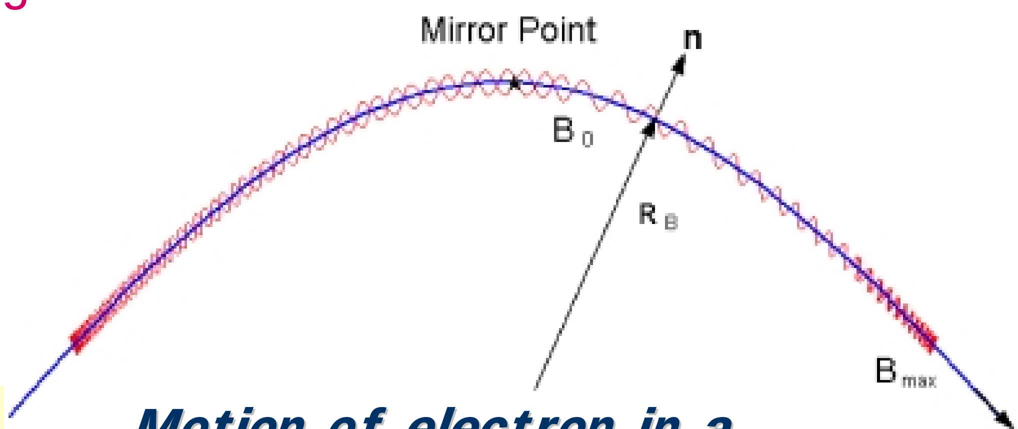
***Field changes slowly***

$$J_\perp = \oint m\upsilon_\perp \rho_s d\varphi = \frac{4\pi m}{e}\mu_m$$

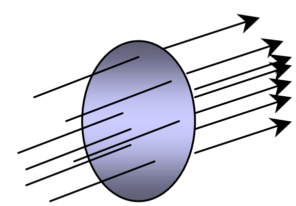
$$J_\parallel = \oint m\upsilon_\parallel dl \qquad \mu_m = \frac{m\upsilon_\perp^2}{2B}$$ is the magnetic moment

$\upsilon_\perp$ $\upsilon_\parallel$ the gyration and parallel velocity

$\rho_s$ the Larmor radius




***Motion of electron in a mirror magnetic field***



## Trapping mechanism — Mirror field trap *(contd.)*

### *Invariation value of motion*

$$W = \frac{m\upsilon^2}{2} = \frac{m\upsilon_\parallel^2}{2} + \frac{m\upsilon_\perp^2}{2} = cons\tan t$$

$$\frac{1}{2} m\upsilon_\parallel^2 + \mu_m B = const$$

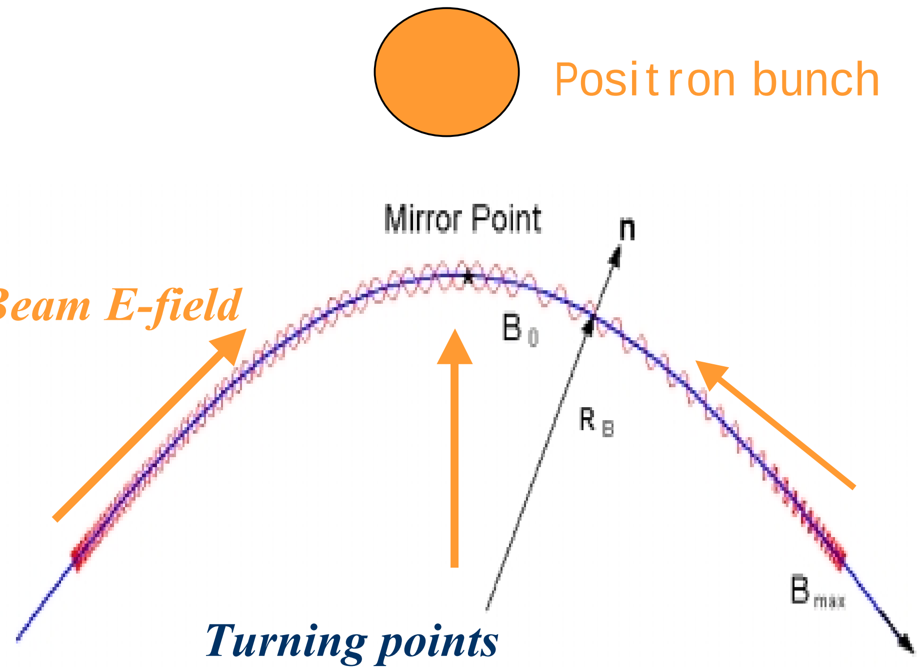
⇒ Reflective Points: $u_\parallel = 0$

### *Trapping condition*

$$\Gamma_{trap} > 1$$

$$\Gamma_{trap} = \frac{F_\upsilon}{F_B} = \frac{\upsilon_{\perp 0}^2}{\upsilon_{\perp 0}^2 + \upsilon_{\parallel 0}^2} \frac{B_{max}}{B_0}$$

***Trap factor is constant if no other force (except B force) disturbs the electron and smaller than 1.0, no trapping***




***mirror field trapping***

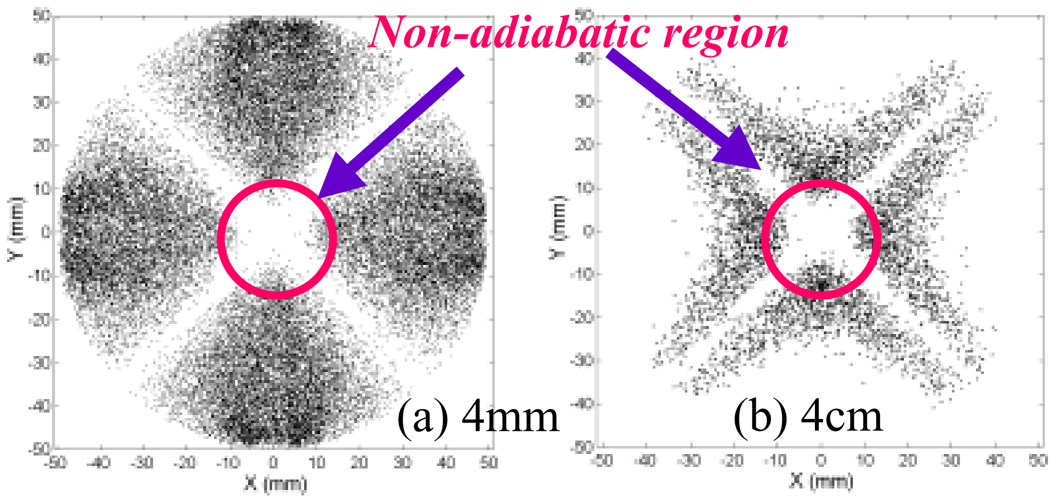
$$\Gamma_{trap} = \frac{\upsilon_{\perp 0}^2}{\upsilon_{\perp 0}^2 + \upsilon_{\parallel 0}^2}\Bigg|_{at\ the\ emission\ po\mathrm{int}} = cons\tan t \le 1$$

# Trapping mechanism ---Beam potential effect on $F_{u,}$ $G_{trap}$

## *Trap requirement for positron bunch*

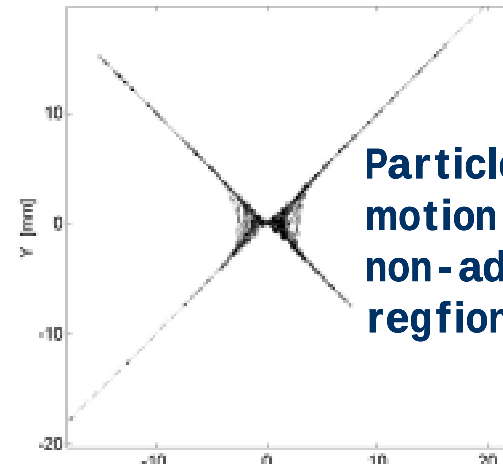
**Bunch length should be shorter than period of gyration motion**

$$\sigma_l < \frac{2\pi cm}{e}\frac{1}{B} \Longrightarrow \sigma_l(mm) < 10.7 / B(T)$$

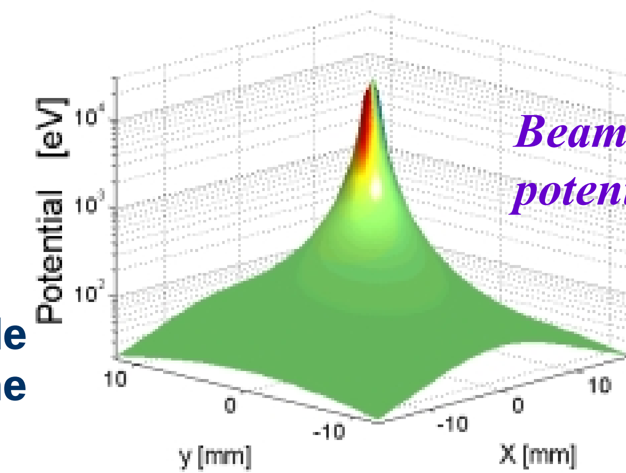



(a) 4mm (b) 4cm

**Trapped photoelectron distribution in quadrupole magnet with field gradient 10.3 T/m during the train gap for different bunch length**

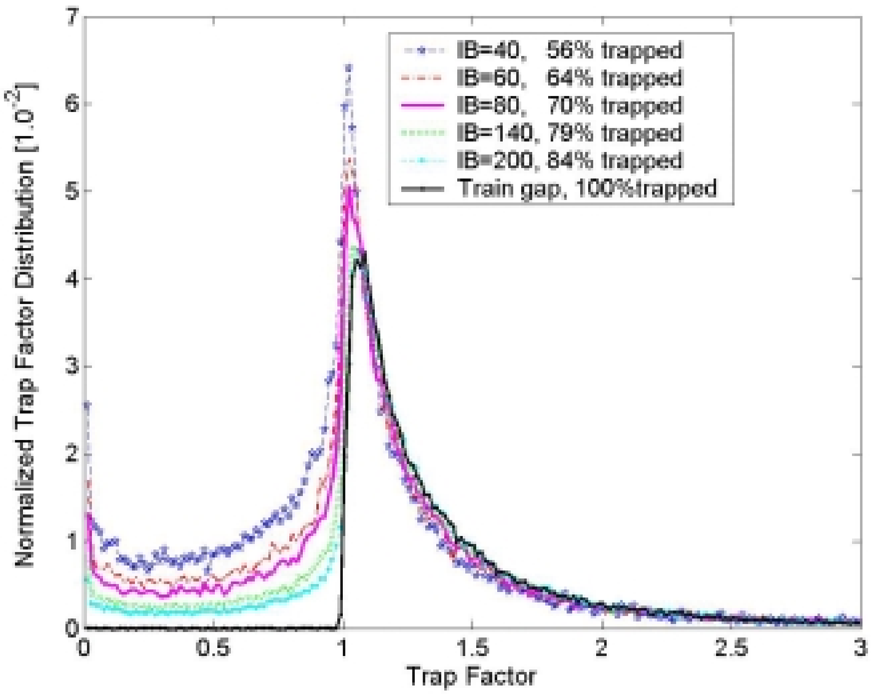



**Particle motion in non-adiabatic regfion**

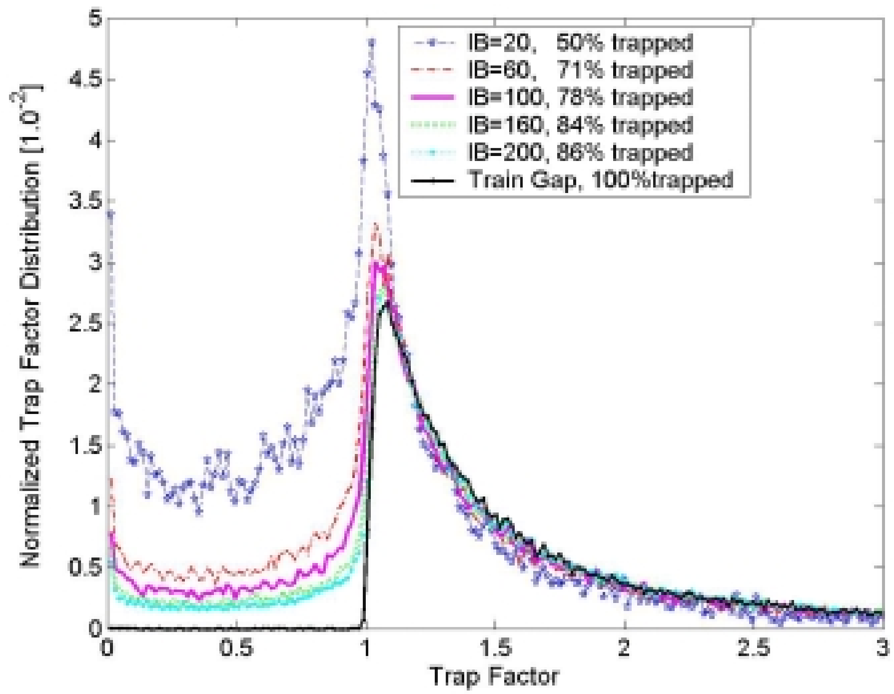



***Beam potential***

## Beam potential effect on trapping —$F_{u}, G_{trap}$




In Quadrupole Magnet



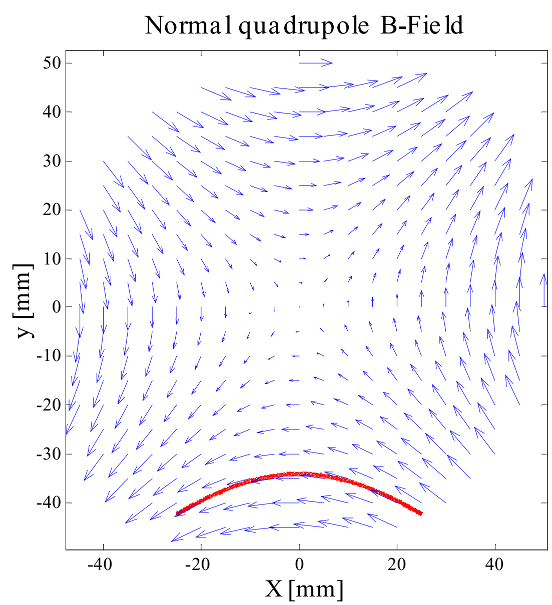

In Sextupole Magnet

***Normalized trap factor distribution of the electron cloud at different time*** **(4mm bunch, 4ns bunch spacing)**

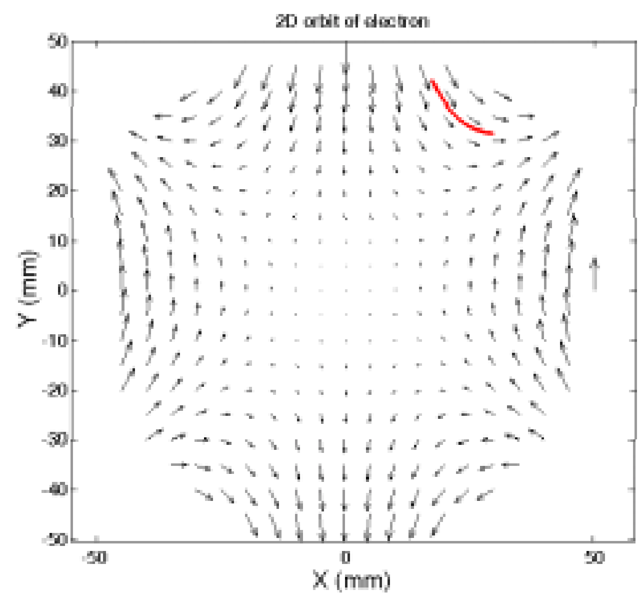
# Orbit of Guiding Center

$$\mathbf{A}^* = \mathbf{A} + \frac{m\upsilon_\parallel}{eB}\mathbf{B} = const.$$ **for general fields**

$$A_z = const.$$ **for translationally symmetric quadrupole & sextupole fields**




$$A_z = A_2(x^2 - y^2)$$




$$A_z = A_3(x^3 - 3xy^2)$$

## Longitudinal Velocity of the Guiding Center (Beam direction)

### •Magnetic gradient drift

$$\bar{\mathbf{v}}_{grad} = \frac{m v_\perp^2}{2eB^3}\mathbf{B}\times\nabla\mathbf{B}$$ With normal gradient $-B\mathbf{n}/R_B$

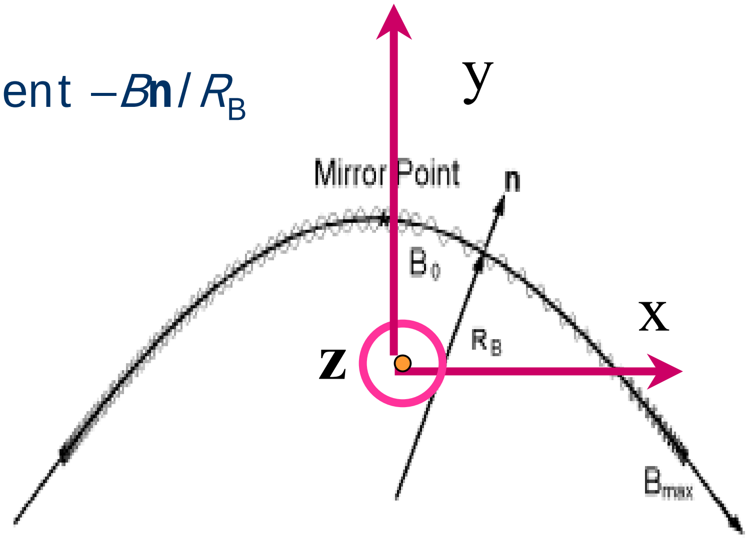
### •Centrifugal force drift

$$\mathbf{F}_c = \frac{m v_\parallel^2 \mathbf{R_B}}{R_B^2} = \frac{m v_\parallel^2}{R_B}\mathbf{n}$$

$$\bar{\mathbf{v}}_F = \frac{\mathbf{F}\times\mathbf{B}}{eB^2} = \frac{\mathbf{n}}{B\Omega_s}\times\frac{\mathbf{B}}{R_B}v_\parallel^2$$

$$\bar{\mathbf{v}}_{gz} = \frac{\mathbf{n}}{B\Omega_s}\times\frac{\mathbf{B}}{R_B}\left(v_\parallel^2 + v_\perp^2/2\right)$$

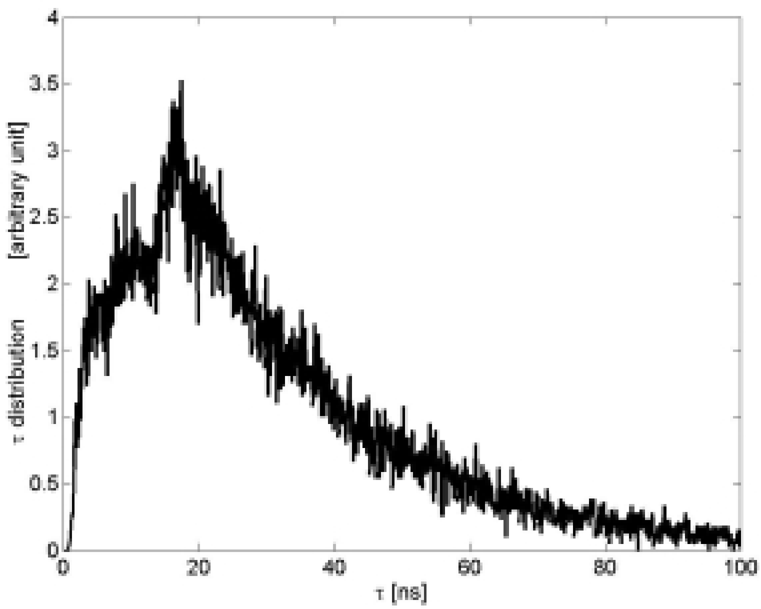
$$\tau = \oint\frac{dl}{v_\parallel} \qquad \bar{\bar{v}}_{gz} = \frac{1}{\tau}\oint\frac{\bar{v}_{gz}}{v_\parallel}dl$$

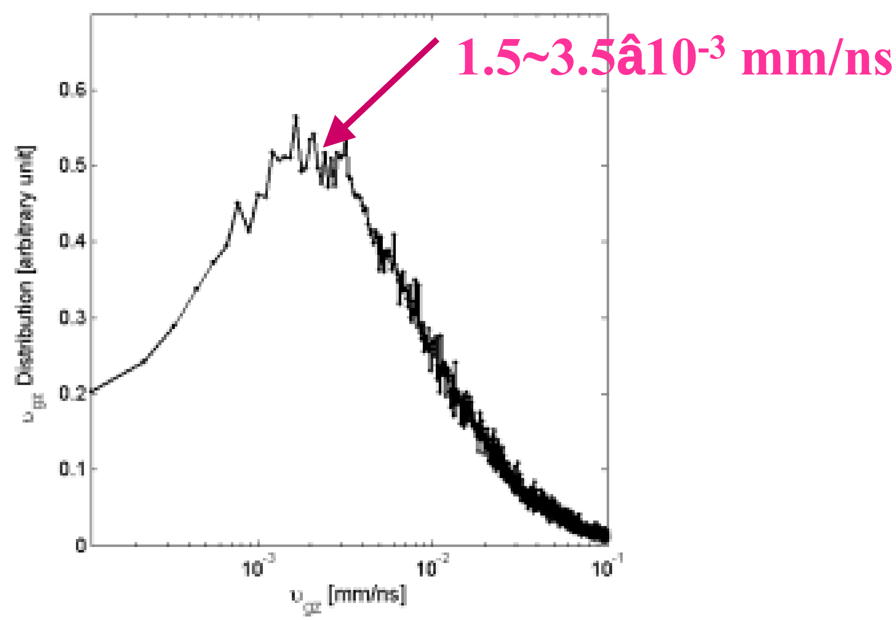



Example: one electron in Quadrupole:

Simulation: 236 ns and 0.0066 mm/ns
Analysis: 228 ns and 0.0063 mm/ns

## Distribution of period and $u_{gz}$



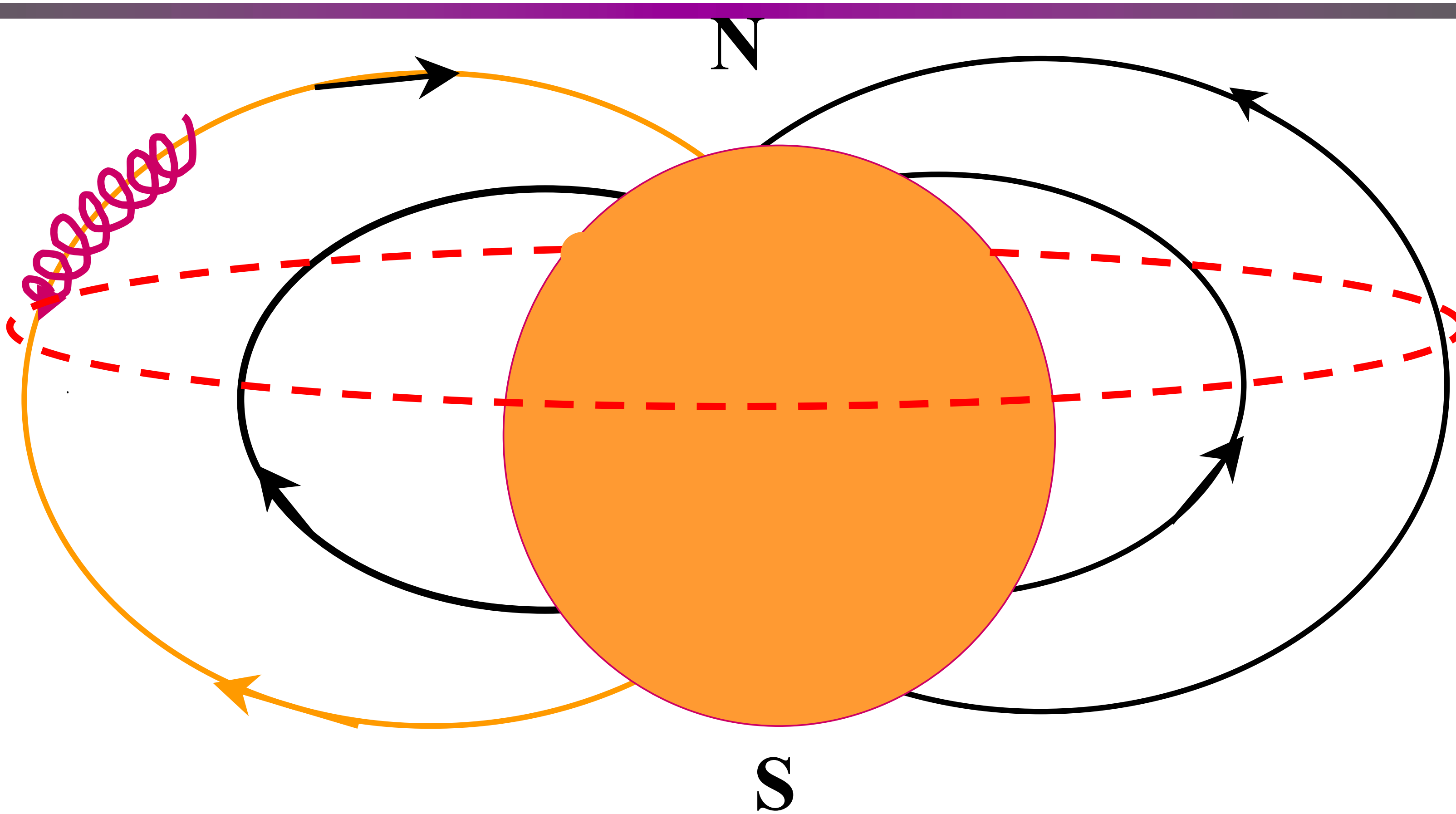




Period **t** and average z-direction drift velocity **$u_{gz}$** of the trapped electron cloud in quadrupole of KEKB LER during the train gap for 4mm bunch length

Magnets length=0.4m + Very low beam direction drift velocity

Long trapping time(~$10^5$ ns) → **Coulped-bunch effects!!**

# Particle trapping in the sky




***Motion of a charged particle in the earth's magnetic field***

## summary

- Mechanism of electron motion in different fields
- Multi-pacting in drift region and dipole magnet, which cause single beam effect due to the high central cloud density and heating of the chamber wall
- The special trapping phenomenon has been found and it happens in quadrupole and sextupole magnets
- The mechanism is mirror field trap, which is caused by the disturbance of positron beam. Theoretical analysis is consistent with simulation very well.
- Trapped electron cloud may cause coupled bunch effects to the positron bunch due to the long trap time

# ACKNOWLEDGMENTS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- Thanks Prof. Kurokawa and Dr. Frank Zimmermann for their help on funding Wang to participate this seminar. Thanks Prof. Kurokawa and Mrs. Hayashi for the great help on the travel to CERN.