Head-Tail Instability Caused by Electron Cloud

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Outline of the talk

Ingredients of the head-tail model

a) Wakefields of electron cloud, its decoherence ⇒ the impedance model

b) Standard formalism, including the machine chromaticity

c) Simplified model of the transverse feedback

d) Combined action of feedback and the chromaticity

Effect of electron cloud pinching

a) Smooth model

b) Single-particle effects

Effect of electron cloud on beam-beam dynamics

Head-tail instability of colliding bunches as a typical two-stream phenomenon

Conclusion

Abstract

The strong head-tail instability of a positron bunch may be caused by wakefields arising in the electron cloud present in the beam pipe. These wakefields are known to produce both deflection and tuneshift varying along the bunch. We discuss a model involving this tuneshift as well as the machine chromaticity and transverse feedback.

Thanks to many colleagues from KEKB, to H. Fukuma, K. Ohmi and F. Zimmermann for collaboration.

Equations of motion

The photoelectron cloud is already present prior to arrival of the bunch whose motion is studied.

Linear equations for the beam centroid offset $y_b(s,t)$, and electron cloud centroid $y_c(s,t)$ at the machine azimuth s at the time t.

Uniform longitudinal density assumed in both the electron cloud and positron bunch

$$\begin{aligned} \left(\frac{1}{c}\frac{\partial}{\partial t}+\frac{\partial}{\partial s}\right)^2 y_b(s,t)+k_0^2\,y_b(s,t) &= g\,\left(y_c(s,t)-y_b(s,t)\right), \\ \frac{\partial^2}{\partial t^2}y_c(s,t) &= \omega_c^2\left(y_b(s,t)-y_c(s,t)\right). \end{aligned}$$

Betatron oscillations are taken in the smooth form with $k_0 = 1/\beta$, β being the vertical amplitude function;

The beam-cloud interaction parameter g

$$g=\frac{4n_c(\pi\sigma_x\sigma_y)\,e^2}{\gamma mc^2\,\sigma_y(\sigma_x+\sigma_y)}=\frac{4\pi n_c r_e\sigma_x}{\gamma(\sigma_x+\sigma_y)},$$

n_c is the time-averaged electron cloud density
σ_y and σ_x are the vertical and horizontal beam sizes,
e is the electron charge,
m is its rest mass,
γ is the beam Lorentz-factor,
c is the speed of light,
r_e is the classical electron radius.

Electrons of the cloud oscillate in the bunch space charge field with the frequency ω_c,

$$\omega_c^2=\frac{4N\,r_e c^2}{\sqrt{2\pi}l\,\sigma_y(\sigma_x+\sigma_y)},$$

here N is the bunch population and l is its Gaussian length.

Equation for the beam centroid alone,

$$\frac{\partial^2}{\partial s^2}y(s,z)+k^2\,y(s,z)=g\frac{\omega_c}{c}\int_0^z dz'\sin\frac{\omega_c}{c}(z-z')\,y(s,z').$$

With a slow-varying complex amplitude $A(s,z)$ of the betatron oscillation,

$$y(s,z)=\mathrm{Re}A(s,z)e^{-iks},$$

after averaging out the A^* term on the right-hand side,

$$\frac{\partial}{\partial s}A(s,z)=i\frac{g}{2k}\frac{\omega_c}{c}\int_0^z dz'\sin\frac{\omega_c}{c}(z-z')\;A(s,z')\,.$$

This corresponds to the beam breakup problem with an oscillating transverse dipole wake function $W(z-z')$,

$$W(z-z')\propto g\frac{\omega_c l}{c}\sin\frac{\omega_c}{c}(z-z').$$

Decoherence. Parametrization of the wake function.

Non-uniformity of the positron bunch density results in the frequency spread of the photoelectron oscillation and thus decoherence of their response. A very simple estimate:

$$W(\omega_c, z) \to \int W(\omega_c(x), z) f(x) dx$$

If we take a Gaussian, $f(x) = \sqrt{\frac{2}{\pi}} e^{-x^2/2}$, $\omega_c(x) = \omega_0 e^{-x^2/4}$. Then,

$$\tilde{W}(z) = g \frac{\omega_0 l}{c} \int_0^\infty \sin\left(\frac{\omega_0 z}{c} e^{-x^2/4}\right) \sqrt{\frac{2}{\pi}} e^{-3x^2/4} dx$$

The result can be fitted either by the Bessel function $J_1(\omega_0 z/c)$, or by the broad-band resonator wake

$$W_1(z) = \frac{cR_S\omega_R}{Q\bar{\omega}} e^{\alpha z/c} \sin\frac{\bar{\omega} z}{c},\ (z < 0),\ \alpha = \frac{\omega_R}{2Q},\ \bar{\omega} = \sqrt{\omega_R^2 - \alpha^2}.$$

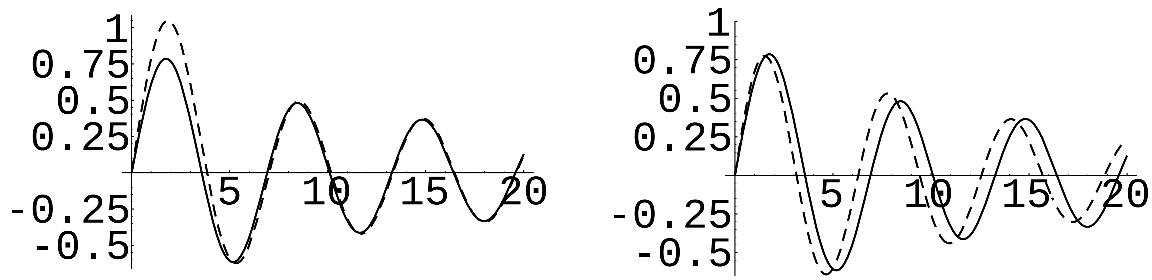

Figure 1: The decoherence wake (solid line) and the fitting function (dashed line). Left: fit by the Bessel function $J_1(\omega_0 z/c)$; right: fit by the broad-band resonator with $\overline{\omega} = \omega_0$.

The corresponding transverse impedance is sampled by the long bunch spectrum in the low-frequency range ($\omega_R l/c \sim 3$),

$$Z_1 = \frac{cR_S/Q}{\dfrac{\omega}{Q} + i\left(\omega_R - \dfrac{\omega^2}{\omega_R}\right)} \approx \frac{cR_S}{Q\omega_R}\left(\frac{\omega}{Q\omega_R} - i\right).$$

The broad-band resonator parameters relevant to the KEKB LER are determined from K. Ohmi's simulations of the wake function.

Table 1: Basic parameters of the KEKB LER

particle type	e^+
circumference	3016 m
beam energy	3.5 GeV
bunch population	3.3×10^{10}
bunch spacing	8 ns
rms beam sizes	0.42 mm
	0.06 mm
bunch length	5 mm
rms energy spread	0.0007
slippage factor	1.8×10^{-4}
chromaticity	$4/8$
synchrotron tune	0.015
betatron tune	≈46
average beta function	15 m

Table 2: Analytically determined parameters for wake force induced by electron cloud using the resonator approximation. R_S/Q in units of Ω is obtained by $cR_S/Q \times 30$. R_S/Q, which linearly depends on ρ_c, and ω_b are evaluated for $\rho_c = 10^{12}$ m^{-3}.

	x	y
$\omega_c[\mathrm{s}^{-1}]$	6.4×10^{10}	1.70×10^{11}
$\omega_b[\mathrm{s}^{-1}]$	1.7×10^5	4.5×10^5
$cR_S/Q[\mathrm{m}^{-2}]$	1.5×10^5	2.9×10^6

Table 3: Simulated parameters for the wake field induced by an electron cloud of density $\rho_e = 10^{12}$ m^{-3}, as obtained by fitting to the resonator model.

	x	y
$\omega_R[\mathrm{s}^{-1}]$	8.7×10^{10}	2.2×10^{11}
Q	2.7	6.3
$cR_S/Q[\mathrm{m}^{-2}]$	2.9×10^6	8.3×10^6

Strong head-tail instability

Notation and formalism of the linearized Vlasov equation analysis follow A. Chao's book:

N_b is the number of positrons in a bunch,

$\rho_{x(y)}(s, z')$ is the horizontal (vertical) dipole moment of particles at z',

η is the slippage factor,

δ is the relative momentum deviation,

$\omega_{\beta,x(y)} = c/\beta_{x(y)}$ is the horizontal (vertical) angular betatron frequency in smooth approximation,

$\xi = \frac{\partial \ln \omega_\beta}{\partial \ln E}$ is the chromaticity,

ω_s is the angular synchrotron frequency.

The distribution function can be split into unperturbed term and perturbation

$$\Psi = \Psi_0 + \Psi_1 e^{-i\Omega s/c} \tag{1}$$

Ψ_0 is expressed via functions of the unperturbed invariants of motion for each degree of freedom,

$\Psi_0 = \psi_0(q)\varphi_0(r)$, where

$q = \sqrt{c^2 p_y^2/\omega_{\beta,y}^2 + y^2}$, $y = q\cos\theta$, $p_y = -\omega_{\beta,y}/cq\sin\theta$

$r = \sqrt{\eta^2 c^2 \delta^2/\omega_s^2 + z^2}$, $z = r\cos\phi$, $\delta = r\omega_s/(\eta c)\sin\phi$.

The Vlasov equation is linearized for a small perturbation of the distribution function, $\Psi_1(q, \theta, r, \phi)$:

$$\left[i\frac{\Omega}{c}\Psi_1 - \frac{\omega_\beta}{c}(1+\xi\delta)\frac{\partial\Psi_1}{\partial\theta} - \frac{\omega_s}{c}\frac{\partial\Psi_1}{\partial\phi}\right] e^{-i\Omega s/c} + \frac{c^2}{\omega_\beta}\frac{\partial\Psi_0}{\partial q}\sin\theta F(z) = 0, \tag{2}$$

where the wakefield action is represented by

$$F(z) = -\frac{N_b r_e}{\gamma}\int W_y(z-z')\rho_{y,1}(s,z')dz' \tag{3}$$

and $\rho_{x(y)}(s, z')$ is the vertical dipole moment of particles at z' for the perturbed distribution Ψ_1.

The solution should be a function of q and θ with a form as follows,

$$\Psi_1 \propto \frac{\partial\Psi_0}{\partial q} e^{i\theta}\sum_{lk} a_{lk} f_{lk}(r) e^{il\phi}.$$

where $f_{lk}(r)$ belongs to a set of orthogonal functions which characterize radial modes and satisfy the normalization

$$\int_0^\infty K(r) f_{lk}(r) f_{lk'}(r) r dr = \delta_{kk'}, \tag{4}$$

where $K(r)$ is the weight function of radial modes. It is related to the unperturbed longitudinal distribution $\varphi_0(r)$:

$$K(r) = \frac{\omega_s}{N\eta c}\varphi_0(r). \tag{5}$$

The problem is reduced to a linear equation set, and Ω results from the corresponding eigenvalue problem,

$$\left(\frac{\Omega-\omega_\beta}{\omega_s}\right) a_{lk} = M_{lk,l'k'}\, a_{l'k'} \tag{6}$$

The matrix M is expressed by

$$M_{lk,l'k'} = l\delta_{ll'}\delta_{kk'} - i\frac{Nr_e c}{2\gamma T_0 \omega_\beta \omega_s} i^{l-l'} \int_{-\infty}^{\infty} Z_1(\omega') g_{lk}(\omega'-\omega_\xi) g_{l'k'}(\omega'-\omega_\xi)\, d\omega' \tag{7}$$

where

$$g_{lk}(\omega) = \int_0^\infty r dr K(r) f_{lk}(r) J_l\left(\frac{\omega}{c} r\right), \tag{8}$$

$\omega' = \omega + \omega_\beta + l\omega_s$, and $\omega_\xi = \xi\omega_\beta/\eta$. The chromatic phase is $\chi = \omega_\xi \sigma/c = \nu_\beta \xi \frac{\sigma_E/E}{\nu_S}$.

The wake force enters via its impedance representation,

$$Z_1(\omega) = i\int_{-\infty}^{\infty} \frac{dz}{c} e^{-i\omega z/c} W(z). \tag{9}$$

For given R_S, Q and ω_R, the broad-band resonator impedance is expressed by

$$Z_1(\omega) = \frac{c}{\omega} \frac{R_S}{1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)}. \tag{10}$$

For a Gaussian distribution in the longitudinal phase space, the unperturbed distribution function and the weight function can be written as

$$\varphi_0(r) = \frac{N\eta c}{2\pi\sigma^2\omega_s} e^{-r^2/2\sigma^2}, \qquad K(r) = \frac{1}{2\pi\sigma^2} e^{-r^2/2\sigma^2}. \tag{11}$$

The orthonormal radial functions are the generalized Laguerre polynomials

$$f_{lk}(r) = \sqrt{\frac{2\pi k!}{(|l|+k)!}} \left(\frac{r}{\sqrt{2}\sigma}\right)^{|l|} L_k^{|l|}\left(\frac{r^2}{2\sigma^2}\right), \tag{12}$$

then

$$g_{lk}(\omega) = \frac{\varepsilon(l)}{\sqrt{2\pi k!(|l|+k)!}} \left(\frac{\omega\sigma}{\sqrt{2}c}\right)^{|l|+2k} e^{-\omega^2\sigma^2/2c^2}, \quad \varepsilon(l) = \begin{cases} 1\,, & l \geq 0\,; \\ (-1)^l, & l < 0\,. \end{cases} \tag{13}$$

This corresponds to the Hermite modes of the dipole moment,

$$\rho_y(z) \propto e^{-z^2/2\sigma^2} H_{|l|+2k}\left(\frac{z}{\sqrt{2}\sigma}\right). \tag{14}$$

We consider azimuthal mode coupling only for two lowest radial mode ($k = 0, 1$).

The coupling matrix consists of 4 blocks,

$$M = \begin{pmatrix} l\delta_{ll'} + M_{l0,l'0} & M_{l0,l'1} \\ M_{l1,l'0} & l\delta_{ll'} + M_{l1,l'1} \end{pmatrix}, \tag{15}$$

$$M_{lk,l'k'} = -i\frac{Nr_e c}{4\pi\gamma T_0\omega_\beta\omega_s}\frac{i^{l-l'}\varepsilon(l)\varepsilon(l')}{\sqrt{k!(|l|+k)!k'!(|l'|+k')!}}\int_{-\infty}^{\infty} Z_1(\omega+\omega_\xi)e^{-\omega^2\sigma^2/c^2}\left(\frac{\omega\sigma_z}{\sqrt{2}c}\right)^{|l|+|l'|+2(k+k')} d\omega\,.$$

The tune of each mode $(\Omega - \omega_\beta)/\omega_s$ is obtained by solving the eigenvalue problem for matrix M. The frequency $\Omega = \omega_\beta \pm l\omega_s$ corresponds to the $\pm l$th synchrobetatron sideband. The matrix has infinite dimension because of $-\infty < l < \infty$.

We truncate the matrix at $-4 \le l \le 2$, and calculate the eigenvalues numerically. To check-up the convergency, we compared with the truncation at $-6 \le l \le 4$.

The eigenvalue or tune of each mode is computed as a function of R_S/Q using ω_R and Q from K. Ohmi's simulation.

Figures show the computed mode tunes vs R_S/Q or the cloud density ρ_c, since R_S/Q is linearly related with it.

The positive slope of all the mode tunes resulting from incoherent effect of the electron cloud (single-particle focusing by the cloud) is equal in all the modes; it is ignored in the following figures.

The sum of decrements for a Gaussian bunch

The sum of eigenvalues is equal to the trace of the mode coupling matrix. Hence, the sum of the synchrobetatron mode decrements

$$\sum_{k,l}\mathrm{Im}\Omega = -\frac{Nr_e c}{2\gamma T_0\omega_\beta\omega_s}\int_{-\infty}^{\infty} d\omega'\mathrm{Re}[Z_1(\omega'+\omega_\xi)]\sum_{k,l} g_{lk}^2(\omega')\,.$$

For a Gaussian bunch,

$$\begin{aligned} 2\pi\sum_{k,l} g_{lk}^2(\omega') &= e^{-\omega^2\sigma^2/c^2}\sum_{l=-\infty}^{\infty}\sum_{k=0}^{\infty}\frac{1}{k!(|l|+k)!}\left(\frac{\omega^2\sigma^2}{2c^2}\right)^{|l|+2k} \\ &= e^{-\omega^2\sigma^2/c^2}\sum_{l=-\infty}^{\infty} I_{|l|}\left(\frac{\omega^2\sigma^2}{c^2}\right) \\ &= e^{-x}(I_0(x) + 2I_1(x) + 2I_2(x) + \ldots) \\ &= e^{-x}e^{x} = 1\,. \end{aligned}$$

Since wakefields are real functions of s, $\mathrm{Re}Z_1(\omega)$ is an odd function of ω with a vanishing average. Thus, the sum of the mode decrements, $\sum_{k,l}\mathrm{Im}\Omega$, also vanishes.

Effect of diffusion on higher-order head-tail modes

With the fast-oscillating wake (or for a “long” bunch), the positive chromaticity can stabilize all the lower-order head-tail modes up to mode numbers $|l| + 2k \le (\omega_R\sigma_z/c)^2$. However their decrements will be compensated by (small) increments of a large number of higher-order modes to give a vanishing sum. But there is a reason why the higher-order modes are of no special concern.

In $e+e-$ machines quantum fluctuations of the synchrotron radiation cause diffusion in particle oscillations. Consider $|l|, k \gg 1$, then the dipole moment is given by the Hermite mode,

$$\rho_y(z) \propto e^{-z^2/2\sigma^2} H_{|l|+2k}\left(\frac{z}{\sqrt{2}\sigma}\right) \longrightarrow \cos\frac{\tilde{l}z}{\sigma}, \text{ for } |z| \le \sigma.$$

In the above, $\tilde{l} = |l| + 2k$. The Green function of diffusion is

$$G(z, z') = \frac{1}{2\sqrt{\pi D t}} \exp\left[-\frac{(z-z')^2}{4Dt}\right]$$

where the diffusion constant $D \sim \sigma^2/\tau$, and τ is the radiation damping time.

After a short time, $t \ll \omega_s^{-1}$,

$$\tilde{\rho}_y(z) \sim \int_{-\infty}^{\infty} \rho_y(z') G(z, z') dz' = \cos\frac{\tilde{l}z}{\sigma} \exp\left[-\frac{\tilde{l}^2 Dt}{\sigma^2}\right] \sim e^{-\tilde{l}^2 t/\tau} \rho_y(z).$$

Thus, while the incoherent damping gives $\tau/\tilde{l}$, the diffusion smear time is even much shorter, $\sim \tau/\tilde{l}^2$.

A simple model of transverse feedback

A bunch-by-bunch feedback integrates the dipole moment over the total bunch length ($\sigma = 5$mm !) and applies its proportional kick after one turn, with a tunable gain and phase shift. The feedback kicker pulse is practically constant over this bunch length. At KEKB $2\pi\nu_s \ll 1$, thus the one-turn delay may not cause a problem like in LEP machine.

Assuming a perfectly linear (no gain saturation) and noiseless feedback hardware, we can describe its action by an equivalent transverse impedance,

$$Z_{FB} = g_{FB} e^{i\phi_{FB}} \delta(\omega)$$

where g_{FB} and ϕ_{FB} are the feedback gain and phase.

The feedback phase parameter can be tuned to purely resistive, $\phi_{FB} = \pi/2$, or purely reactive, $\phi_{FB} = 0,\ \pi$, or mixed mode.

At zero chromaticity, the feedback only acts upon the $l = 0$ mode; at positive chromaticity, higher-order synchrobetatron modes are also influenced.

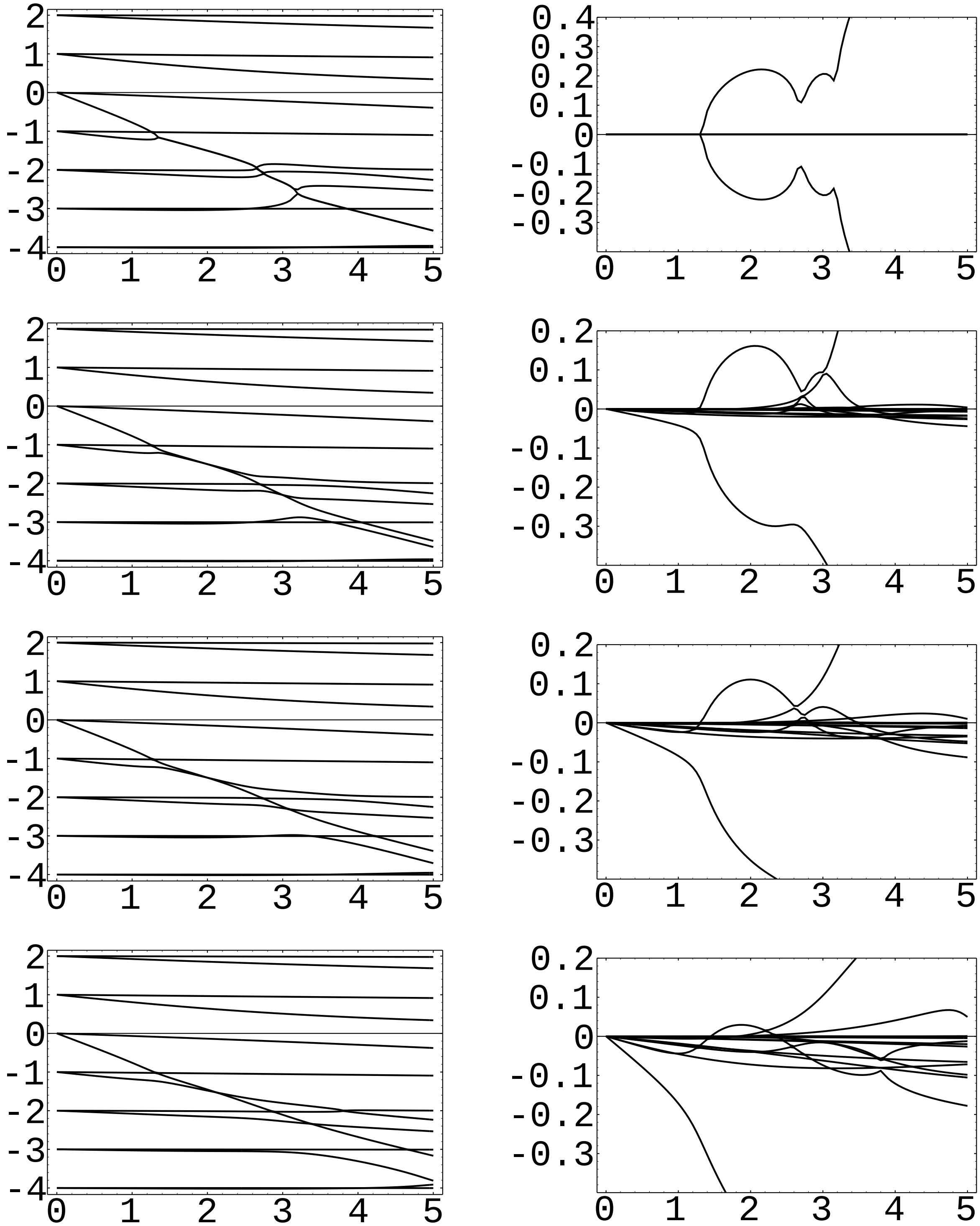

Figure 2: Head-tail mode tunes in units of the synchrotron tune vs the cloud density $\rho_c \times 10^{-12}$m^{-3}. Left: real part, right: imaginary part. From top to bottom: the chromatic phase is 0.0, 0.125, 0.25, 0.5; $Q = 1$, $I_b = 0.52$mA.

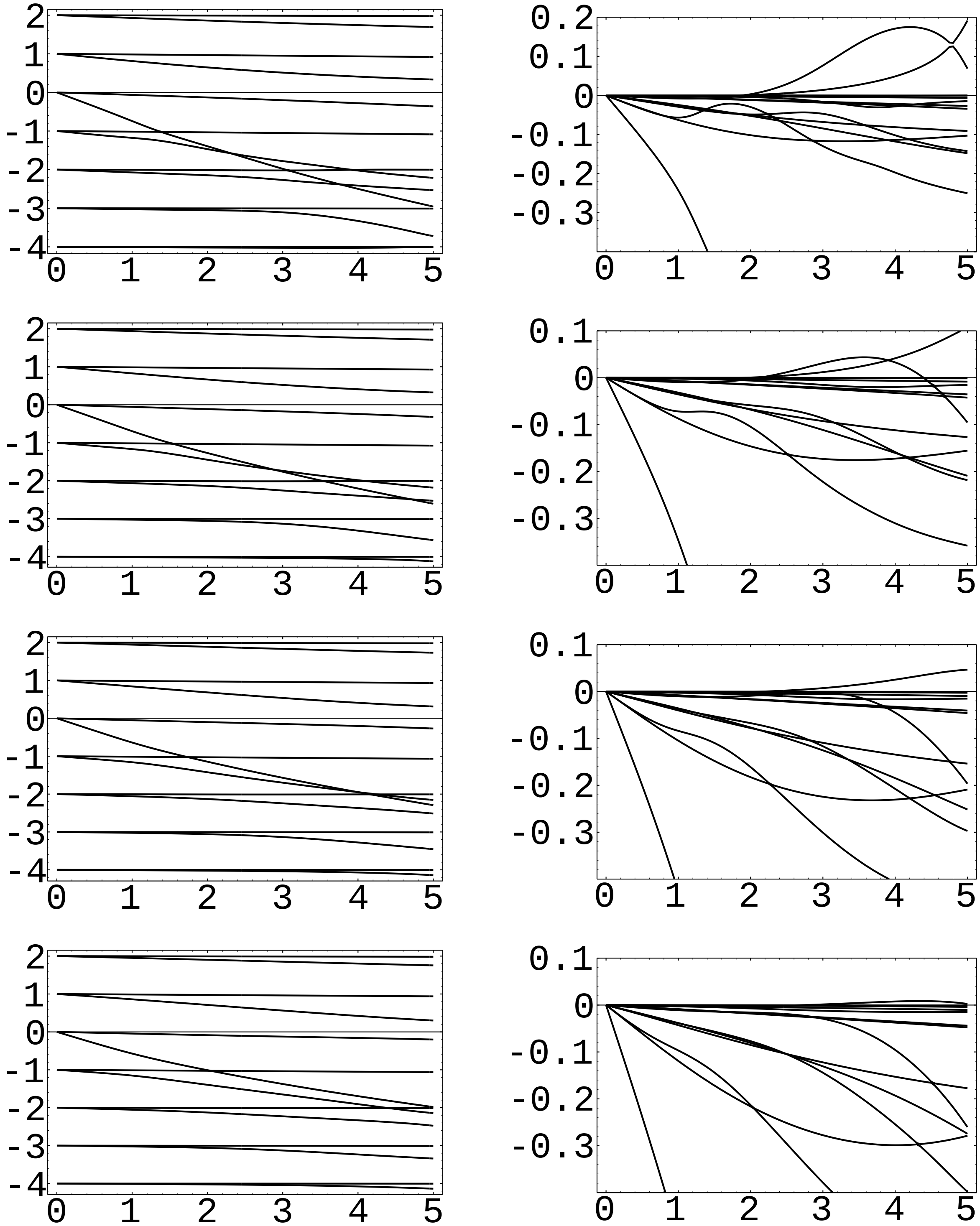

Figure 3: Head-tail mode tunes in units of the synchrotron tune vs the cloud density $\rho_c \times 10^{-12}\mathrm{m}^{-3}$. Left: real part, right: imaginary part. From top to bottom: the chromatic phase is 0.7, 1.0, 1.25, 1.5; $Q = 1$, $I_b = 0.52$mA.

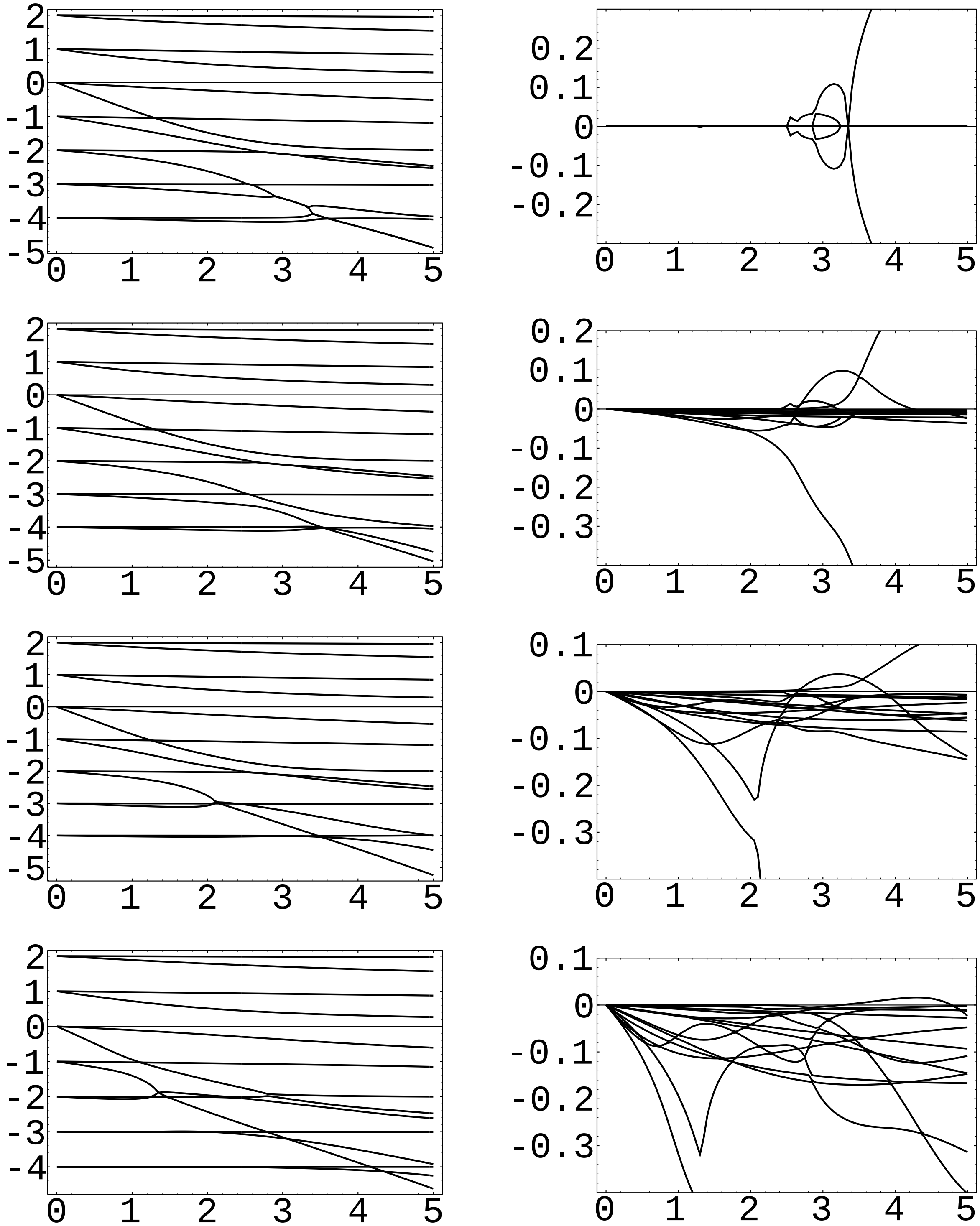

Figure 4: Head-tail mode tunes in units of the synchrotron tune vs the cloud density $\rho_c \times 10^{-12}\mathrm{m}^{-3}$. Left: real part, right: imaginary part. From top to bottom: the chromatic phase is 0.0, 0.125, 0.5, 1.0; $Q = 6.3$, $I_b = 0.52$mA.

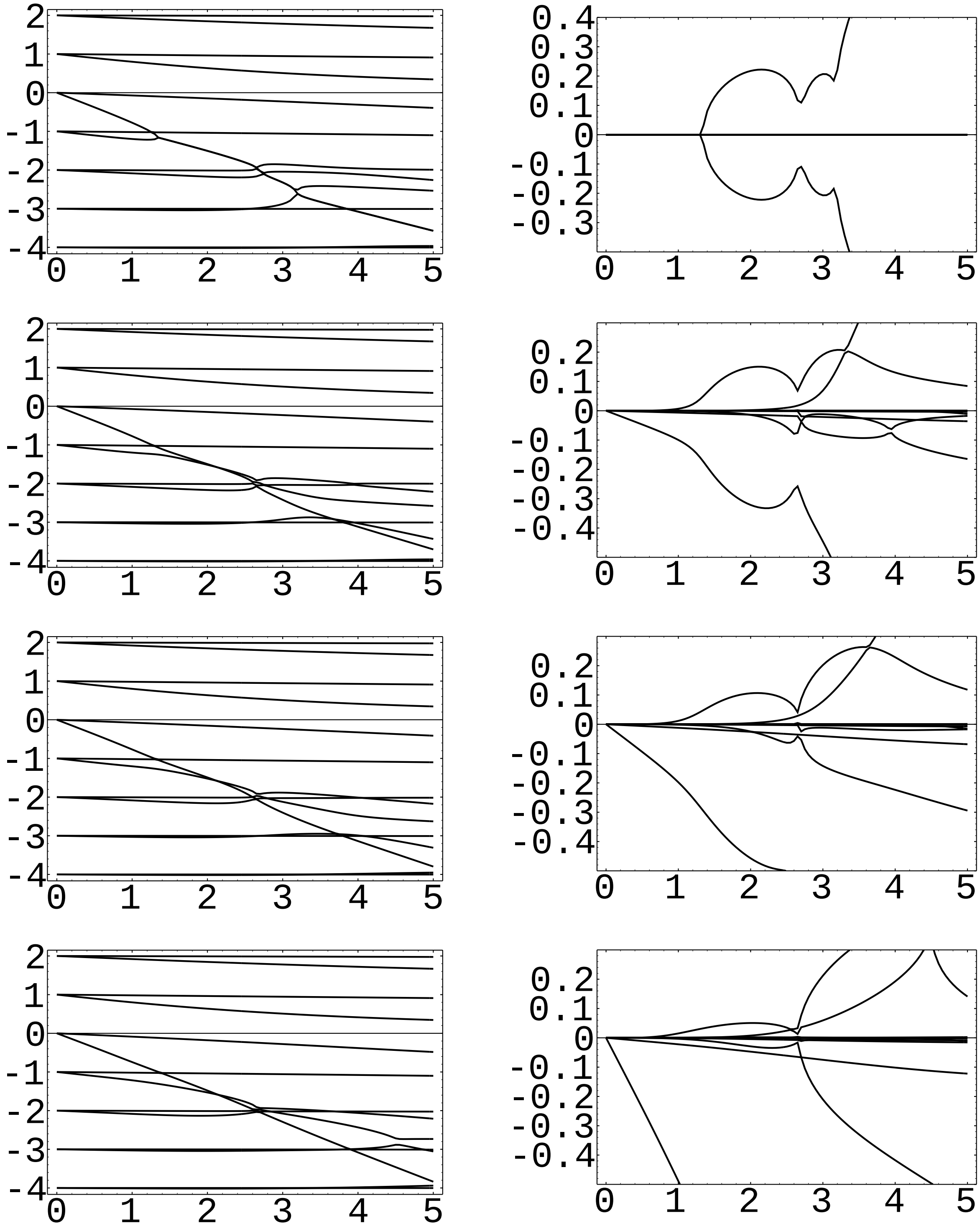

Figure 5: Head-tail mode tunes in units of the synchrotron tune vs the cloud density $\rho_c \times 10^{-12}\mathrm{m}^{-3}$. Left: real part, right: imaginary part. From top to bottom: the feedback damping is 0.0, 0.1, 0.2, 0.5; $Q = 1$, $I_b = 0.52$mA and zero chromaticity.

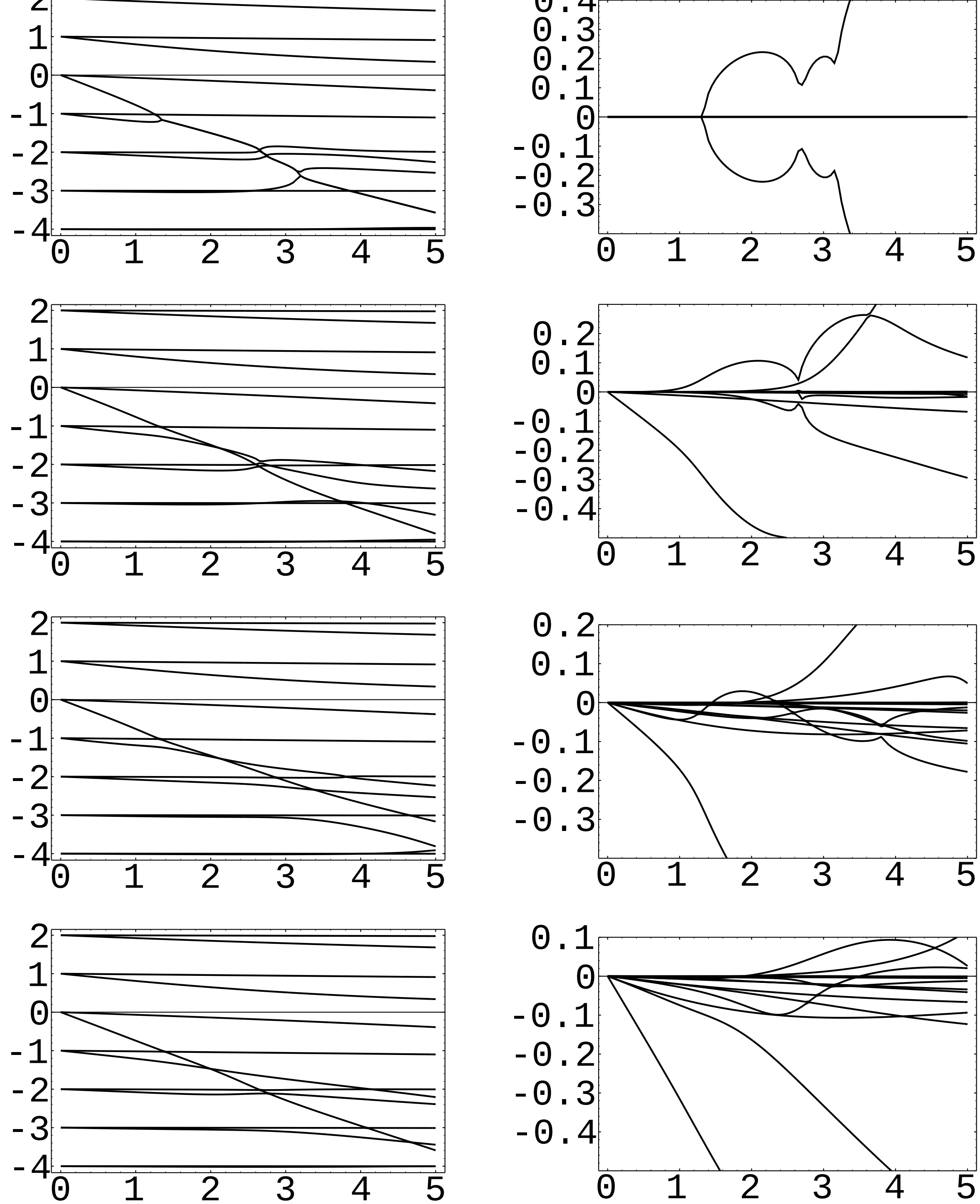

Figure 6: Head-tail mode tunes in units of the synchrotron tune vs the cloud density $\rho_c \times 10^{-12}$m^{-3}. Left: real part, right: imaginary part. Combined action of the chromaticity and feedback: the feedback damping is 0.2, and the chromatic phase is 0.5. $Q = 1$, $I_b = 0.52$mA.

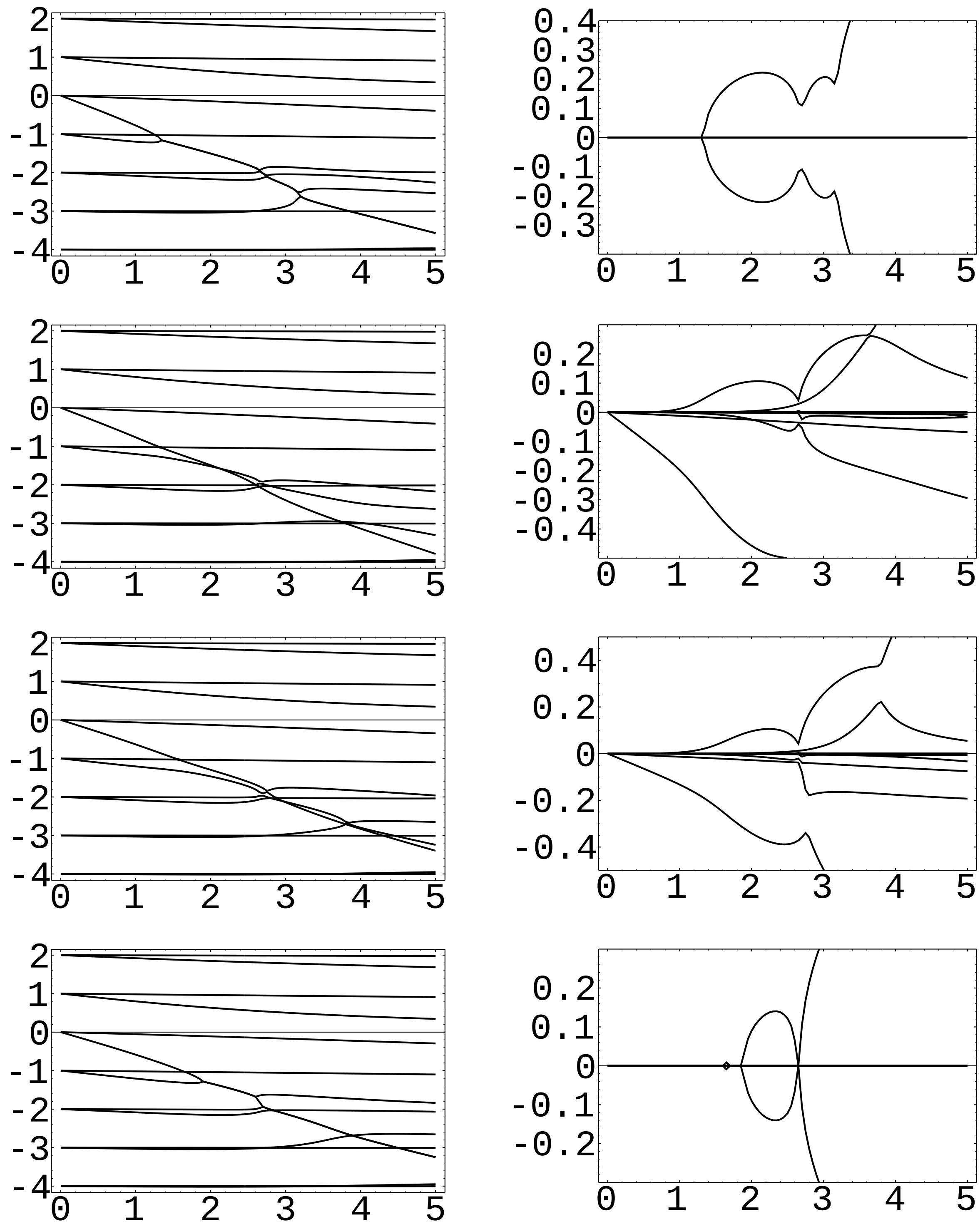

Figure 7: Head-tail mode tunes in units of the synchrotron tune vs the cloud density $\rho_c \times 10^{-12}\mathrm{m}^{-3}$. Left: real part, right: imaginary part. Effect of the feedback phase: the feedback damping is 0.2, and its phase is varied 90^o, 135^o, 180^o. $Q = 1$, $I_b = 0.52$mA.

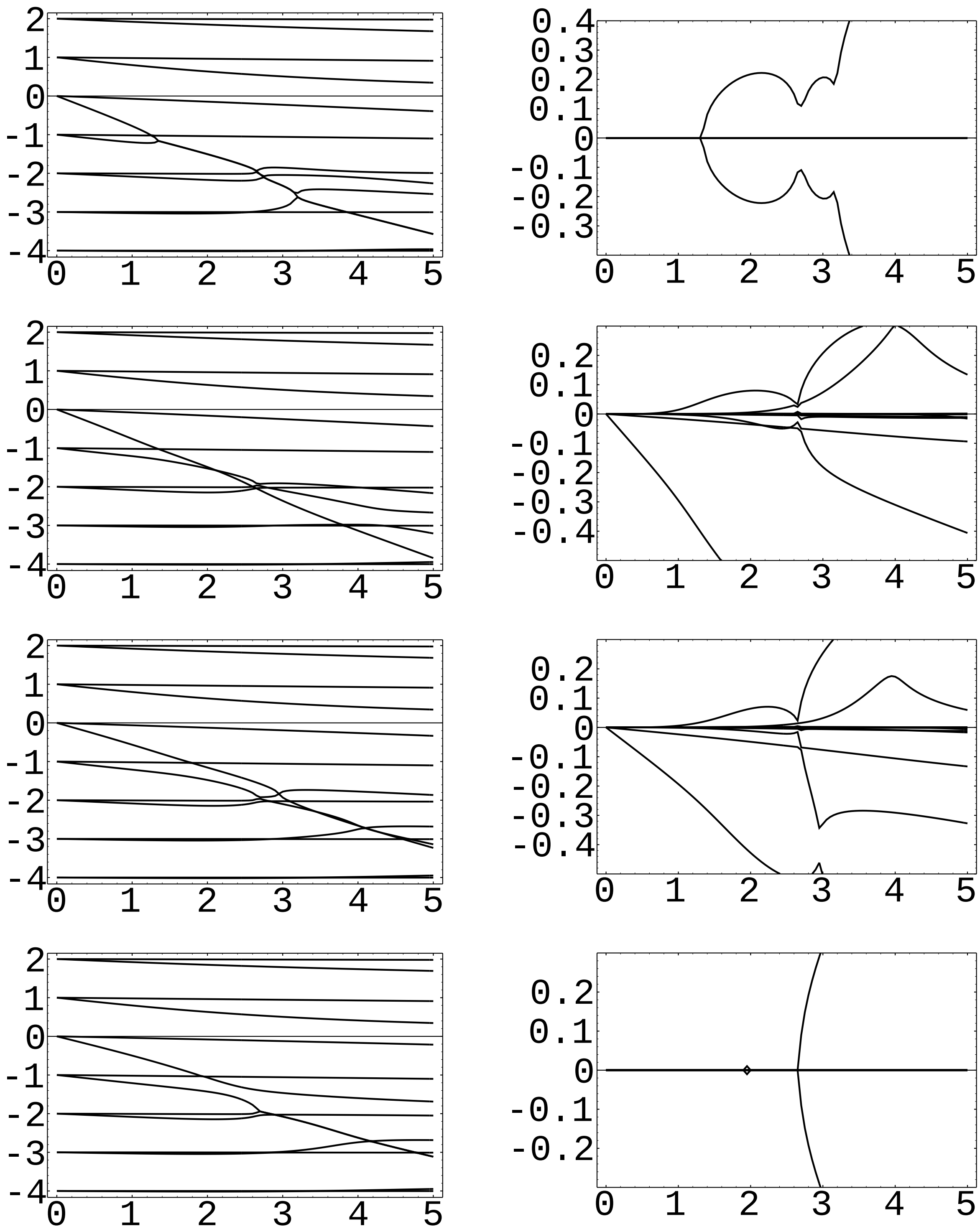

Figure 8: Head-tail mode tunes in units of the synchrotron tune vs the cloud density $\rho_c \times 10^{-12}\mathrm{m}^{-3}$. Left: real part, right: imaginary part. Effect of the feedback phase: the feedback damping is 0.3, and its phase is varied 90^o, 135^o, 180^o. $Q = 1$, $I_b = 0.52$mA.

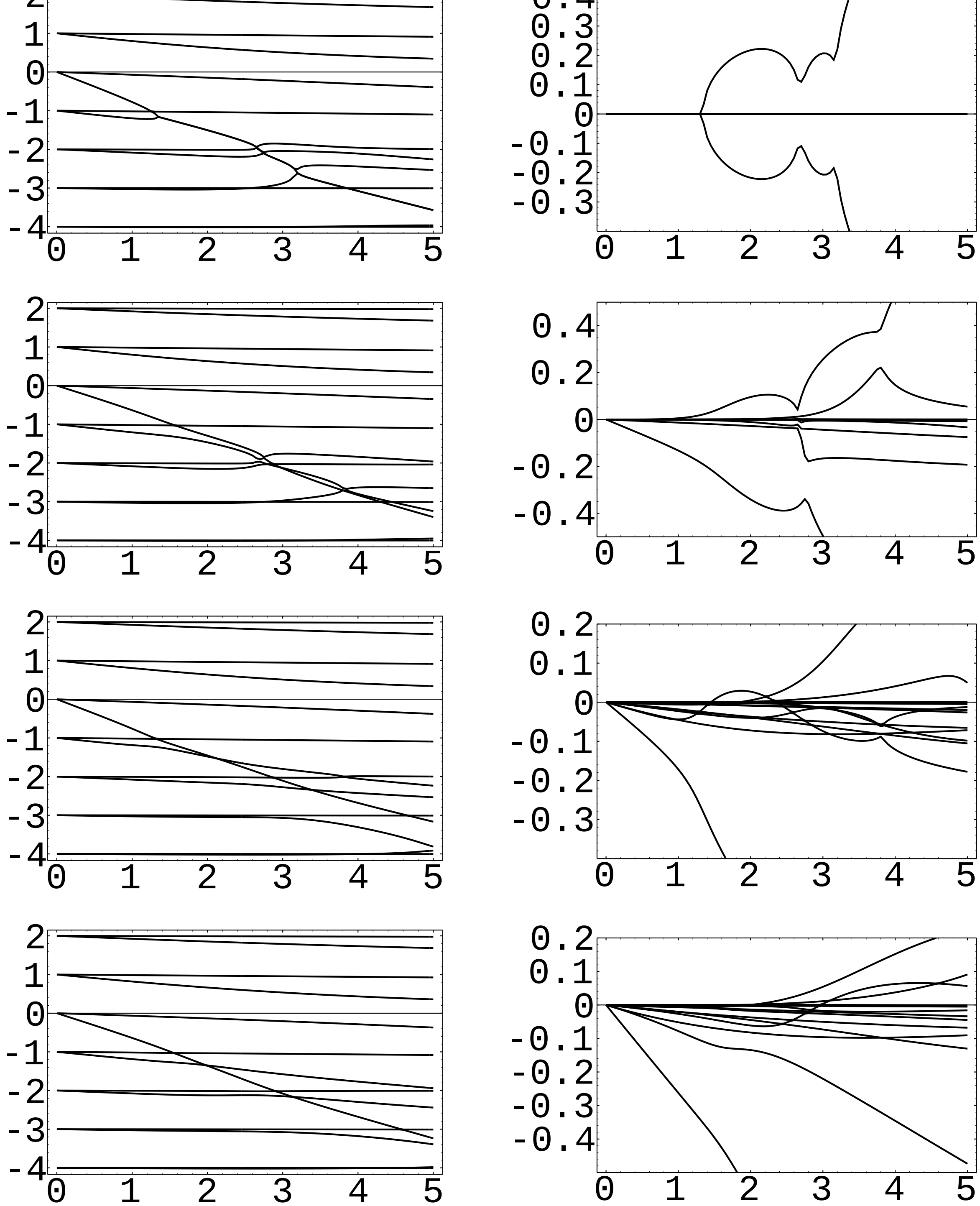

Figure 9: Head-tail mode tunes in units of the synchrotron tune vs the cloud density $\rho_c \times 10^{-12}\text{m}^{-3}$. Left: real part, right: imaginary part. Combined action of the chromaticity and feedback: the feedback damping is 0.2, its phase is 135^o and the chromatic phase is 0.5. $Q = 1$, $I_b = 0.52$mA.

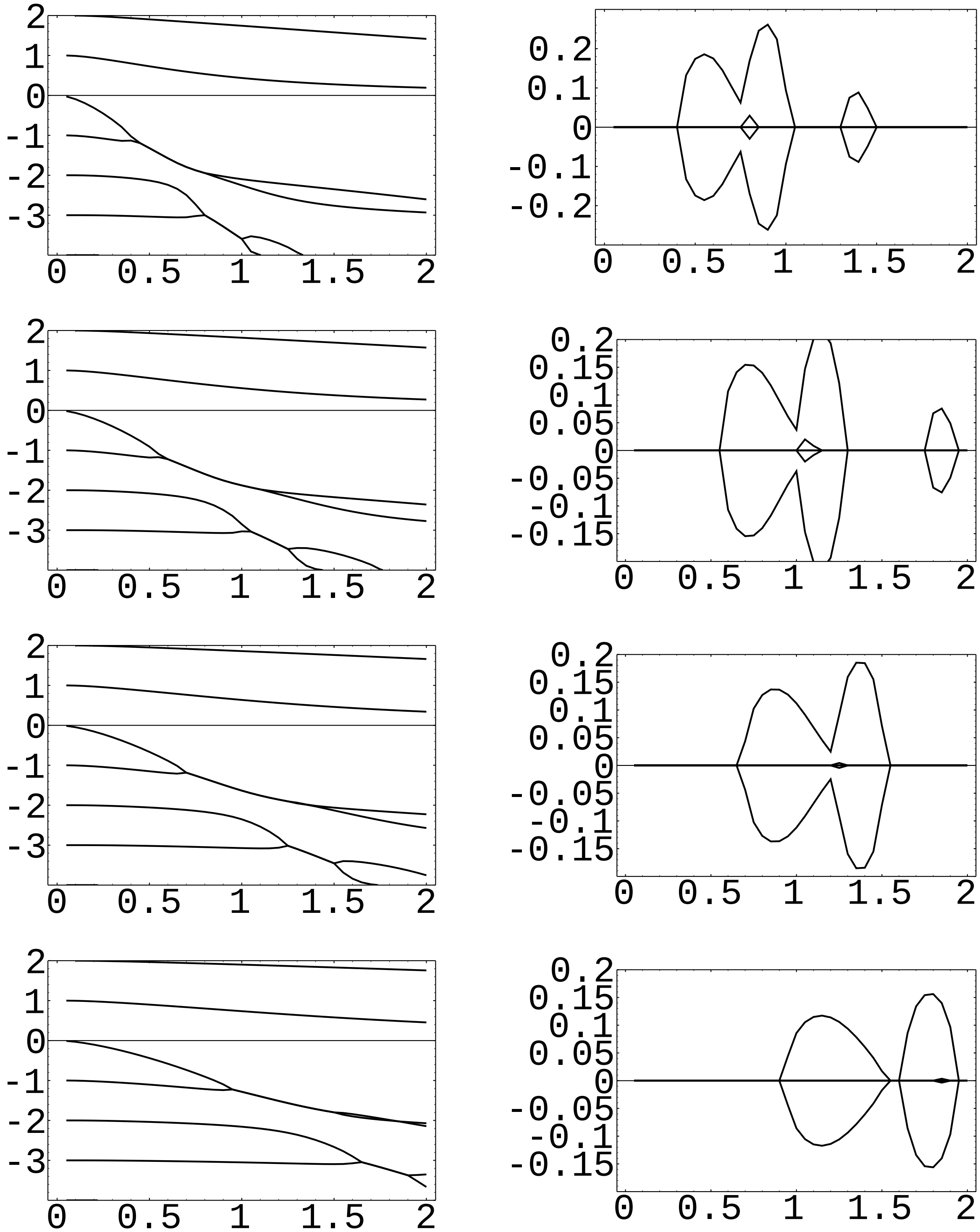

Figure 10: Head-tail mode tunes in units of the synchrotron tune vs the bunch current, mA. Left: real part, right: imaginary part. From top to bottom: the bunch spacing is 2, 3, 4, 6; $Q = 1$.

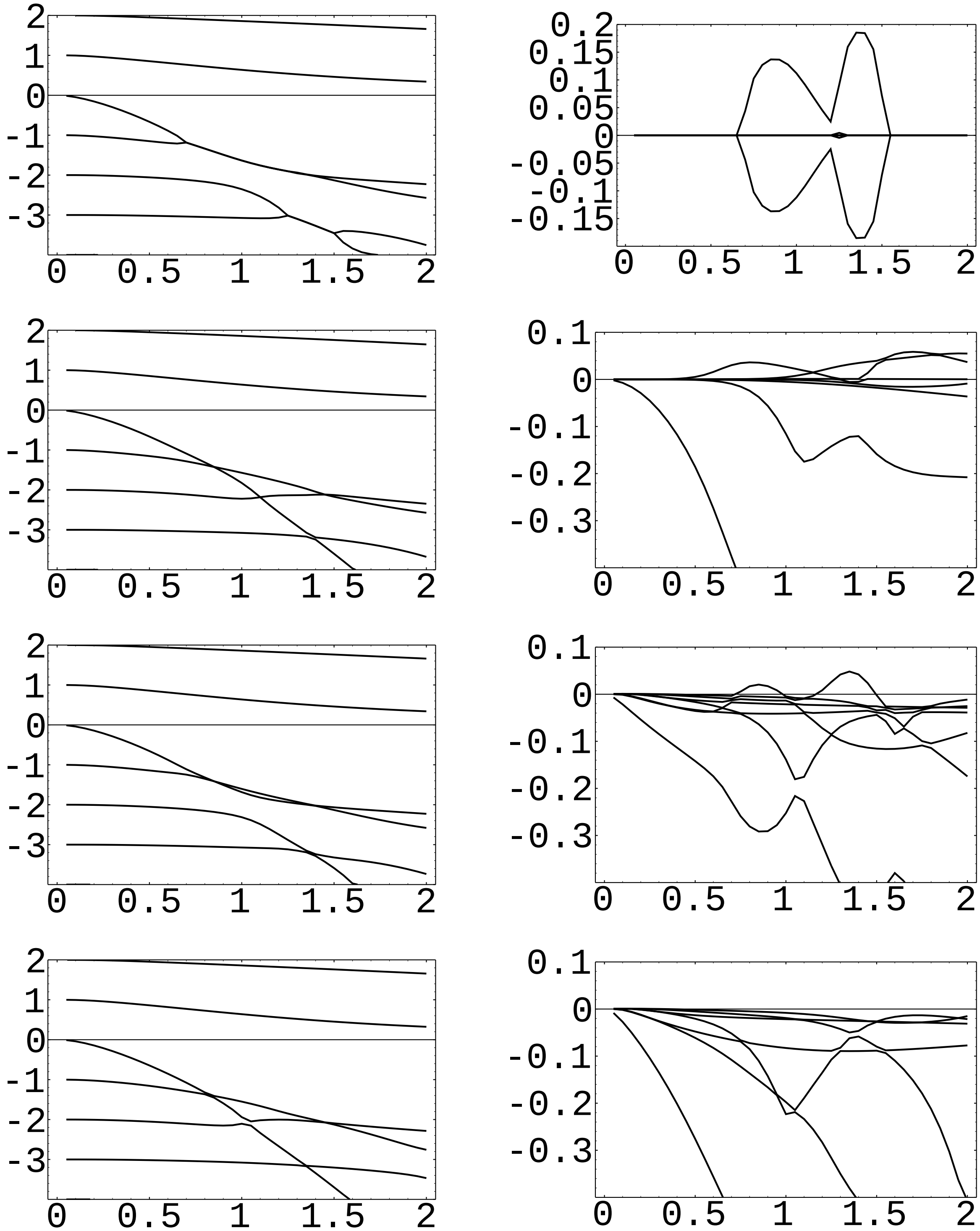

Figure 11: Head-tail mode tunes in units of the synchrotron tune vs the bunch current, mA. Left: real part, right: imaginary part. Combined action of the chromaticity and feedback: the feedback damping is 0.2, and the chromatic phase is 0.5. $Q = 1$

Instability thresholds vs the wake and beam parameters

At zero chromaticity we find the mode 0 and -1 coupling threshold as a function of the electron oscillation frequency. Zotter's estimate gives here $\rho_c \propto \omega_s \omega_R \sigma_z$ in the $\omega_R \to \infty$ limit.

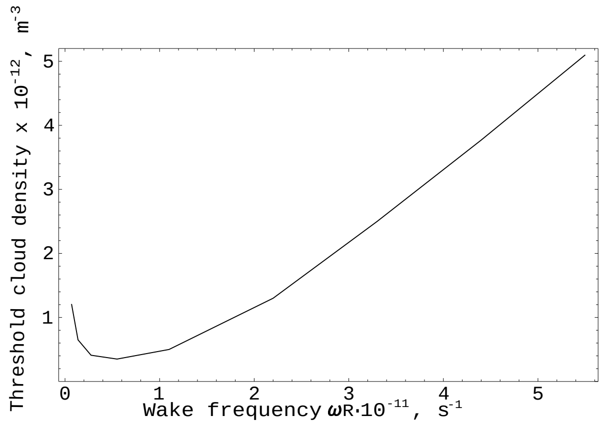

Figure 12: Threshold density of the electron cloud vs the wake frequency for the 4 bucket spacing fill pattern, at fixed bunch current of 0.52mA. $Q = 1$

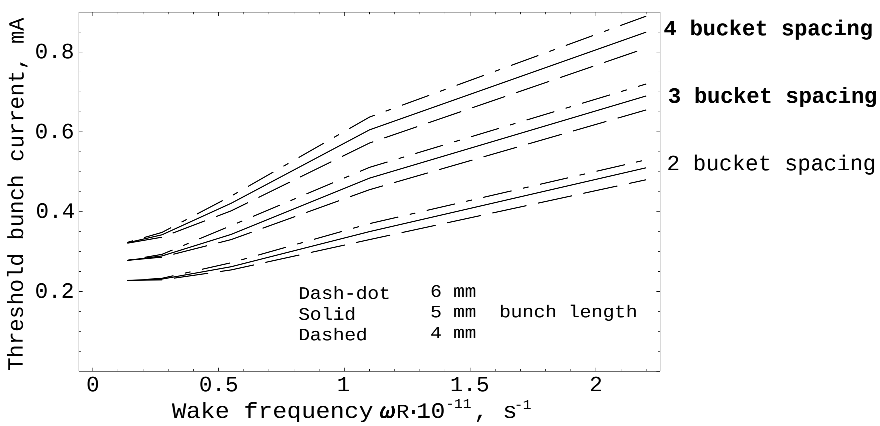

Figure 13: Threshold bunch current $I_{b,th}$ vs the wake frequency for different fill patterns, with cloud densities proportional to the linear charge density,

$$\rho_c = 10^{12} \frac{I_b}{0.52\text{mA}} \frac{4}{L_{sep}} \text{m}^{-3}.$$

Threshold scaling parameter:
at $\omega_R = 2.2 \cdot 10^{11}$ the blowup threshold scales as $\sim I_b^{1.36}/L_{sep}$;
at $\omega_R = 1.1 \cdot 10^{11}$ the blowup threshold scales as $\sim I_b^{1.27}/L_{sep}$.

Mode Stability in the Coasting Beam Limit, $\omega_R\sigma_z/c \gg 1$

Usually one takes the maximum of $\mathrm{Re}Z_1$ to be sure that **all the modes** are stable. For the BBR $\omega_{max} \approx \omega_R$, this means $\mathrm{Re}Z_1(\omega_R)$.

However, for our case with low mode numbers, $l \sim 1$, and $lc/\sigma_z \ll \omega_R$, this will yield too strong a condition (sufficient, but not necessary).

Let us take the coasting-beam limit condition for stability in its **full form**, see Eq. (6.263) in Chao's book:

$$\frac{N_b}{\sqrt{2\pi}\sigma_z} \to \frac{N}{cT_0}, \qquad -\frac{N}{cT_0}\frac{r_e c^2}{2\gamma T_0\omega_\beta}\mathrm{Re}Z_1(n\bar{\omega}_0+\omega_\beta) < \Delta\delta|-n\bar{\omega}_0\eta+\xi\omega_\beta|$$

where $\Delta\delta$ corresponds to σ_E/E for Gaussian bunches.

Relating this coasting beam situation to the bunched beam parameters, we replace

$$\begin{aligned} n\bar{\omega}_0 &\to \omega \ \text{ of the mode} \\ \xi\omega_\beta/\eta &\to \omega_\xi, \ \text{ the chromatic frequency} \\ c\eta\Delta\delta/\omega_s &\to \sigma_z, \ \text{ the bunch length} \end{aligned}$$

For the higher-order modes the accurate treatment by TMC theory shows stability.

For the lower-order modes, $l \sim 1-2$, we take

$$\omega \simeq -\frac{c}{\sigma_z}l \ (\ll \omega_R)$$

and approximate the impedance

$$\mathrm{Re}Z_1 \approx \frac{cR_S}{Q}\frac{\omega}{Q\omega_R^2},$$

then neglecting $\omega_\beta \ll \omega$, we obtain the stability condition for the lth mode

$$\frac{Nr_e c^2}{2\gamma T_0^2\omega_\beta\omega_s\sigma_z}\frac{cR_S}{Q}\frac{1}{Q\omega_R^2} < \left|1+\frac{\omega_\xi\sigma_z}{c\,l}\right| = |1+\chi/l|$$

where χ is the chromatic phase. Note that $\omega_R^2 \propto N_n/\sigma_z \propto N$, $cR_S/Q \propto \rho_c$, and $\omega_s\sigma_z \propto \eta$.

Hence, we come to the scaling of the threshold level of the electron cloud density

$$\rho_{c,\mathrm{th}} \propto \eta\, Q\, |1+\chi/l|, \quad \text{for } l \sim 1-2.$$

For the case $\omega \approx -\omega_R$, we approximate

$$\mathrm{Re}Z_1 \approx \frac{cR_S}{Q}\frac{Q}{\omega_R}$$

and arrive at somewhat different stability condition,

$$\frac{Nr_e c^2}{2\gamma T_0^2\omega_\beta\omega_s\sigma_z}\frac{cR_S}{Q}\frac{Q}{\omega_R^2} < |1+\omega_\xi/\omega_R|\,,$$

whence the threshold scaling is

$$\rho_{c,\text{th}} \propto \frac{\eta}{Q}|1+\chi\frac{c}{\omega_R\sigma_z}|, \quad \text{for } l \sim \omega_R\sigma_z/c, \text{ i.e., the oscillation parameter.}$$

In the saturation condition, $\rho_{c,\text{th}} \propto N_{b,\text{th}}/L_{\text{sep}}$.

For the opposite situation, $\omega_R\sigma_z/c \ll 1$, from the single-bunch interaction parameter we find a different scaling of the instability threshold,

$$Z_1 N_b/\sigma_z \propto N_b^2/\sigma_z L_{\text{sep}}$$

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Combination of Beam-Beam Interaction and Machine + eCloud Impedance

A precursor work by K. Cornelis for the case $\sigma_z \ll \beta^*$,

K. Cornelis, in Proc. 1994 Chamonix Workshop, CERN Report SL/94-06, p. 185 (1994), the set of synchrobetatron modes was introduced in the beam-beam system.

Two-stream interaction of colliding bunches must be included when $\sigma_z \simeq \beta^*$.

Linear theory and simulation:

[1] E.A. Perevedentsev, “Simplified theory of the head-tail instability of colliding bunches,” in Proc. 1999 Particle Accelerator Conf., New York, 1999, vol. 3, p. 1521 (1999).
[2] E.A. Perevedentsev, “Possible coherent beam-beam instability of the head-tail type,” in Proc. Int. Workshop on Performance Improvement of Electron-Positron Collider Particle Factories, Tsukuba, 1999, KEK Proceedings 99-24, p. 171.
[3] E.A. Perevedentsev and A.A. Valishev, “Simulation of the head-tail instability of colliding bunches”, Phys. Rev. ST-AB **4**, 024403 (2001).

Experiment and simulation:

[4] I.N. Nesterenko, E.A. Perevedentsev and A.A. Valishev, “Coherent synchrobetatron beam-beam modes: experiment and simulation”, Phys. Rev. E, this May.

Predictions of the linear theory

1) Stability of **all** head-tail modes when impedance = 0.
2) Impedance $\Rightarrow$ onset of head-tail instability of colliding bunches, the phase shift from head to tail comes from the betatron phase advance over the interaction length; no threshold; the growth rate

$$\tau^{-1} \approx \frac{\pi\xi_{\text{bb}}\Delta Q_{\text{coh}}}{T_0 Q_s}\sin(2\sigma_z/\beta^*)\,.$$

3) The chromaticity can stabilize only some of the modes.

A 3D beam-beam simulation with “soft” bunches is in order to show the limitation of instability growth due to the nonlinear beam-beam force.

Conclusion

There is a number of indirect evidence in favour of the single-bunch mechanism of the LER vertical blowup, such as reasonable agreement of estimated thresholds with those observed, scaling of the thresholds with the bunch train parameters, the role of the chromaticity and its estimated values needed to stabilize the blowup., etc.

However, direct evidence of the strong head-tail instability means that its signatures have to be observed, such as

1) "banana" oscillations of the bunch $\Leftarrow$ streak-camera, optical detector of coherent oscillations based on a fast photodiode;

2) the synchrobetatron sideband spectrum with its characteristic dependence on current and chromaticity $\Leftarrow$ gated tune meter and Bunch Oscillation Recorder data taken vs the relevant machine parameters, bunch current and charge density, synchrotron tune, chromaticity, feedback settings, short and long bunch trains, etc.

Electron cloud pinching causes the tune variation along the bunch; single-particle SBR's should be kept in mind when analyzing an apparent "instability" by tracking.

Combined action of electron cloud and beam-beam interaction may present a serious limitation of the high-luminosity performance.

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