On the Transparency of the Electron Cloud to Synchrotron Radiation

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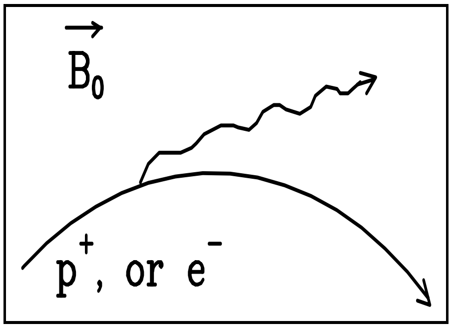
Can the synchrotron radiation, produced by a relat. part. in a bending magnet interact with the electron cloud present in the same magnet?

- **transverse waves only**
- **only beam and a cloud (no vac. chamber)**
- **part radiates as in vacuum, but radiation decays as it traverses the cloud**

A beam particle traverses bending magnet



remote observer (r) sees spontaneous (as in vac.) rad. (energy/sec/$d\omega/d\Omega$ at angle θ w.r.t. B_0) "Schott formula"

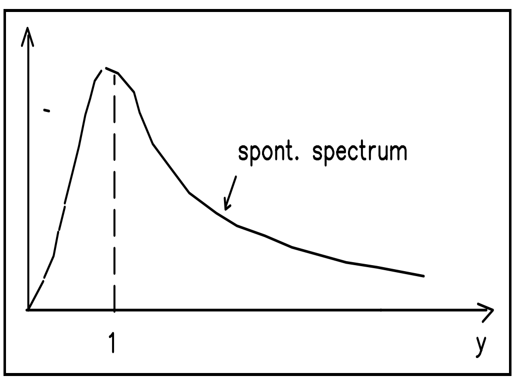
$$r^2 S^0_\omega = (\dots) \sum_{n=-\infty}^{+\infty} (j^2_{\pi,n} + j^2_{\sigma,n}) \delta(\omega - n\Omega)$$

$$(\dots) = \frac{\omega^2 (Ze)^2}{4\pi c^3 \gamma^2}$$

$$tot.\ lost\ ener. = 2\pi \int_0^\infty d\omega \int_0^\pi d\theta \sin\theta \ r^2 S^0_\omega$$

$$j_{\pi,n} = c\gamma \cot\theta \ J_n(n\beta \sin\theta) \qquad \pi$$

$$j_{\sigma,n} = c\beta\gamma \ J'_n(n\beta \sin\theta) \qquad \sigma$$



$y \equiv \frac{2}{3} \frac{\omega}{\Omega \gamma^3}$

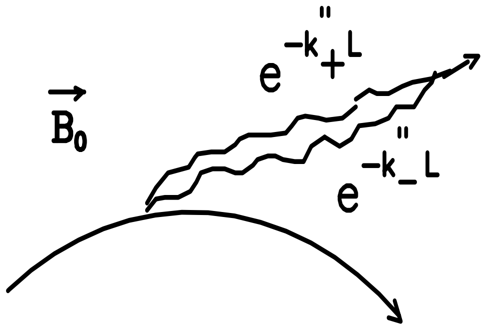
$\Omega = \frac{ZeB_0}{Mc\gamma}$

$M, Ze =$ mass and charge of beam part.

$$J_n(z) = \frac{\epsilon^{1/2}}{\pi\sqrt{3}} K_{1/3}(\eta), \qquad J'_n(z) = \frac{\varepsilon}{\pi\sqrt{3}} K_{2/3}(\eta)$$

etc ...

There is electron plasma within the magnet



remote observer sees different radiation (within plasma the plane wave splits)

L = cloud thickness; θ= angle w.r.t. B_0

- $\omega, k = \omega/c$ splits into ordinary (-) and extraord. (+) waves, (+) rotates in the same dir. as plasma e^-
- Dispersion relation approximations:
  - rare plasma: 1) only these two modes exist; 2) neglect effects at cloud border
  - $L >> \lambda = 2\pi c/\omega \Rightarrow$ take infinite volume
  - high frequencies: $\Rightarrow$ take collisionless and neutralised plasma $\Rightarrow$ standard disp. rel. (as in works on thermo-nuclear reactor)

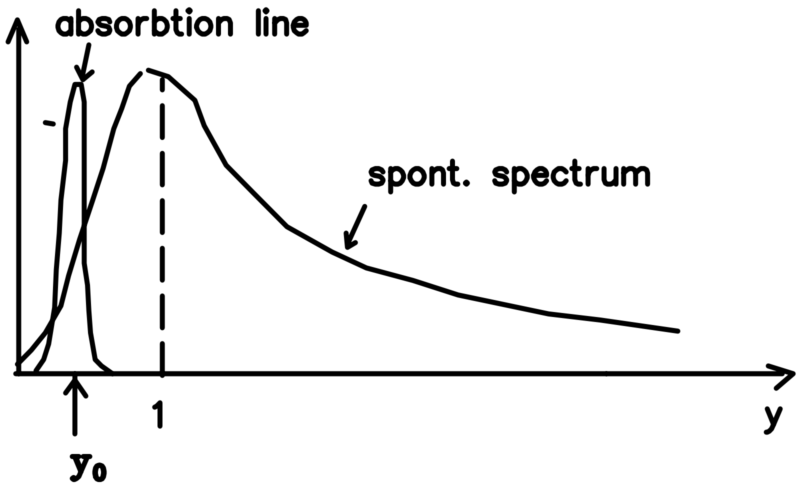
  disp relation $D(\omega, k) = 0 \Rightarrow k_\pm(\omega, \theta) =?$

- Absorption coefficients $k''_\pm(\omega, \theta) = Im\ k_\pm$

One expects exp. fact. in the corrected Schott

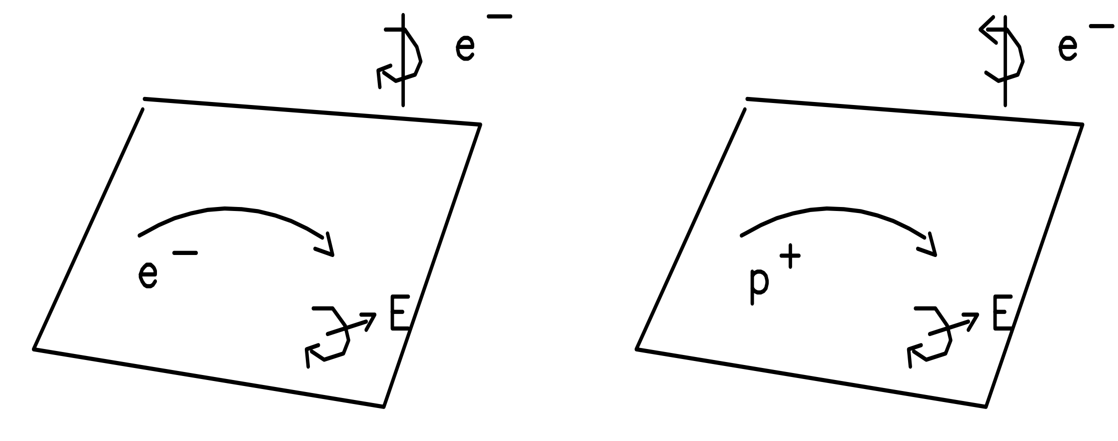
$$\boxed{e^{-k''_{\pm}(\omega,\theta)L} \approx \quad 1 \quad - \quad k''_{\pm}(\omega,\theta)L} \tag{1}$$

as in vac. - absorbed



An estimation for the strength of interaction: the deposited in the cloud fraction of total power radiated

$$2\pi \int_0^\infty d\omega \int_0^\pi d\theta \sin\theta \;\times k''_{\pm}(\omega,\theta)L$$



projection of E vector of rad. on the median plane

rotates in the same dir. as the radiating particle

- **If the beam particle is e^-, then the projection of E rotates the same direction as the electrons**

  $\Rightarrow$ stronger interaction for e^- beams propagating inside e^- plasma (?)

  $\Rightarrow$ weaker in the realistic case of a cloud (?)

parameters

B_0 ext. magn. field; L cloud thickness

N_0 numb. of el. per cubic cm

$\omega_p = \left(4\pi N e^2/m_e\right)^{1/2}$ el. plasma freq.

$\Omega_e = \frac{eB_0}{m_e c}$ el. cycl. freq.

$\Omega = \frac{ZeB_0}{Mc\gamma}$ cycl. freq of beam part.($\gamma >> 1$);

$q = (\omega_p/\Omega_e)^2$ plasma density par.;

v_e r.m.s. thermal vel. for Maxwellian distr.

$f_e(v) = N_0 \left(\frac{m_e}{2\pi T_e}\right)^{3/2} e^{-mv^2/2T_e}$

$\beta_e \equiv v_e/c;\quad T_e \equiv m_e v_e^2$

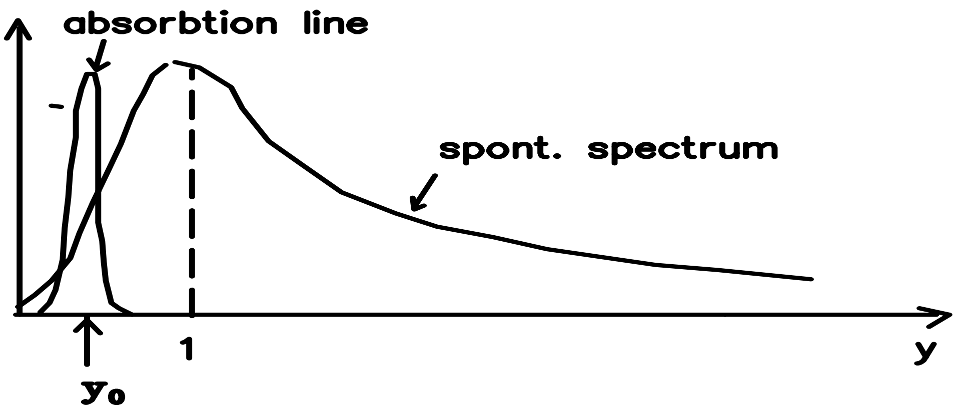
Rough estimation for Maxwellian plasma

$$\frac{q}{\beta_e}\frac{L}{\lambda}\ \ y_0\ \ \beta_e\ \sim 10^{-7}$$

$L = 10\ m;\quad \lambda = 1\ mm;$
$N_0 = 10^6\ cm^{-1};\quad q = 10^{-8};\quad y_0 \sim 10^{-3}$

The distance of the electronic gyro-frequency to the crit. frequency of the spont. spectrum

$$\boxed{y_0 \equiv \frac{2\Omega_e}{3\Omega\gamma^3} = \frac{2}{3\gamma^2}\ \frac{M}{Z\ m_e}}. \qquad (2)$$



$y_0 << 1$ for any machine

1. for positron rings
   $y_0 \sim \gamma^{-2}$

2. For protons in the LHC
   – injection $\gamma = 480,\ y_0 = 1/200$
   – collision $\gamma = 7500,\ y_0 = 2 \times 10^{-5}$.

Radiation as in vacuum (Schott form.)

$$r^2 S^0_\omega = (\dots) \sum_{n=-\infty}^{+\infty} (j^2_{\pi,n} + j^2_{\sigma,n}) \delta(\omega - n\Omega)$$

$$j_{\pi,n} = c\gamma \cot\theta \; J_n(n\beta \sin\theta)$$
$$j_{\sigma,n} = c\beta\gamma \; J'_n(n\beta \sin\theta)$$

Radiation with absorption

$$r^2 S_\omega = (\dots) \sum_{n=-\infty}^{+\infty} \delta(\omega - n\Omega) \frac{1}{1+\cos^2\theta} \times$$
$$(j_{\pi,n}\cos\theta + j_{\sigma,n})^2 e^{-2k''_+ L} + (j_{\pi,n} - j_{\sigma,n}\cos\theta)^2 e^{-2k''_- L}$$

- for $e^{-2k''_\pm L} = 1 \quad \Rightarrow$ as in vac.
- $\cos\theta$ factors appear because these are scalar products (squared) between $\vec{j}_n$ and the eigen-vectors $\vec{e}_+$ and $\vec{e}_-$ of the extraord. and ord. modes
- this formula is for e^- beam in e^- cloud
- for e^+, p^+ beam in e^- cloud we exchange $(+)$ and $(-)$
- and expand $e^{\cdots}$

Absorbed part

$$r^2 S_\omega = (\dots)\sum_{n=-\infty}^{+\infty}\delta(\omega - n\Omega)\frac{1}{1+\cos^2\theta}\times$$

$$\left[(j_{\pi,n}\cos\theta + j_{\sigma,n})^2\ k''_{\pm}L + (j_{\pi,n} - j_{\sigma,n}\cos\theta)^2\ k''_{\mp}L\right]$$

A : upper sign $\Rightarrow$ e^- in e^- cloud

B : lower sign $\Rightarrow e^+(p^+)$ in e^- cloud

For Maxwellian cloud near the first e^- cycl. res::

$$k''_+(\omega,\theta)L \approx \frac{\Omega_e^2}{\omega}\frac{qL}{\beta_e c}\ \frac{1+\cos^2\theta}{\cos\theta}\ e^{-\left(\frac{\omega-|\Omega_e|}{\sqrt{2}\omega\beta_e\cos\theta}\right)^2} \sim \frac{qL}{\beta_e\lambda};$$

$$k''_-(\omega,\theta)L\ \sim \frac{qL}{\lambda}\ e^{-(\dots)}\qquad \boxed{(\lambda\equiv 2\pi c/\omega;\ \omega = n\Omega\approx\Omega_e)}$$

Conclusions (made before int. is carried out)

- take observer above med. plane and $\omega > 0$ then n, cosθ, $j_{\pi,n}$ $j_{\sigma,n}$ are all positive
- k''_+ is $\frac{1}{\beta_e}\sim 100$ times larger $\Rightarrow$ we only keep extraord. (+) wave
  $\Rightarrow$ in case A only first term remains
  $\Rightarrow$ in case B only second term remains
- for the first term $j_{\sigma,n}$ and $j_{\pi,n}$ add up, hence in case A stronger absorption
- B: absorpt. = zero in med. plane ($\sim$ cosθ)

Maxwellian cloud at frequencies near the first el. cyclotron resonance

plasma density parameter $q = (\omega_p/\Omega_e)^2$

temperature 100 eV $\beta_e \equiv \bar{v}_e/c \sim 0.01$

$$\left(\frac{\omega_n}{\omega}\right)^2 \frac{1}{\beta_e} = \frac{q}{n^2\beta_e} << 1$$

is fulfilled for all n **of** $\omega = n\ \Omega_e$, **even for** $n = 1$

The plasma polarization tensor

$$Q = \begin{pmatrix} -\frac{q}{4} + \sigma & \frac{iq}{4} + i\sigma & 0 \\ \frac{-iq}{4} - i\sigma & -\frac{q}{4} + \sigma & 0 \\ 0 & 0 & -q \end{pmatrix} \qquad (\sigma >> q)$$

$$\sigma = i\sqrt{\frac{\pi}{8}} \frac{\omega_p^2}{\omega^2 \beta_e \cos\theta}\ w(z_1),$$

$$w(z_1) = e^{-z_1^2}\left(\frac{\cos\theta}{|\cos\theta|} + \frac{2i}{\sqrt{\pi}}\int_0^{z_1} e^{y^2} dy\right)$$

- $w(z)$ **is the complex error funct.**
- $z_n \equiv \frac{\omega - n|\Omega_e|}{\sqrt{2}\omega\beta_e\cos\theta} \sim 0$ **for n=1 and** $>> 1$ **for other** n
- **the effect caused by the presence of resonance electrons, for which** $\omega - |\Omega_e| \approx (v_\parallel/c)\ \omega\cos\theta$

Case B: estimation of the second term

integration over angles and frequencies (here $W_0 y_0 = \gamma\Omega_e/\rho$):

$$\frac{\Delta W}{W_0} = \frac{\sqrt{\pi}}{4\sqrt{2}}\ \frac{(Ze)^2\ \Omega_e^2\ qL}{\rho\ c\ \beta_e} 2\pi \int_0^\infty d\omega \int_0^\pi d\theta \sin\theta\ \times$$

$$\times \sum_{n=1}^{\infty} n|\cos\theta| \left(\frac{J_n}{\sin\theta} - J_n'\right)^2\ \boldsymbol{e}^{-\left(\frac{\omega-|\Omega_e|}{\sqrt{2}\omega\beta_e\cos\theta}\right)^2} =$$

$$= \frac{\sqrt{\pi}}{2\sqrt{2}} \frac{1}{3\pi^2} \left(\frac{3}{2}\right)^4 \frac{\Omega_e}{c}\ \frac{qL}{\beta_e}\ \ y_0\ \gamma\ \times$$

$$\int_0^\infty y dy \int_0^{+\gamma} d\psi\psi \left[\sqrt{1+\psi^2} K_{1/3}(\eta) - \frac{1+\psi^2}{\gamma} K_{2/3}(\eta)\right]^2 e^{-\boldsymbol{z}^2}$$

$$\approx 0.3\ \frac{\Omega_e}{c}\ q\ L\ \ y_0^{4/3},$$

where

$$\boldsymbol{z} = \frac{n\ \Omega - \Omega_e}{\sqrt{2}\ n\ \Omega\ \beta_e\ \ \cos\theta} = \frac{y - y_0}{\sqrt{2}\ y\beta_e\psi/\gamma} = \frac{1}{\psi\Delta}\left(1 - \frac{y_0}{y}\right),$$

$$\Delta \equiv \sqrt{2}\beta_e/\gamma, \qquad y_0 = \frac{2\Omega_e}{3\Omega\gamma^3}$$

$$\boldsymbol{\frac{\Delta W}{W_0} \approx 0.3\ \frac{\Omega_e}{c}\ q\ L\ y_0^{4/3}}$$

scales as: $\sim \gamma^{-8/3}$ $\qquad \sim B_0^{-1}$

Fraction of deposited energy with LHC parameters for the case of Maxwellian velocity distr.

	p^+ **collision**	p^+ **injection**
γ	**7460**	**480**
B_0	**83860**	**5390**
$\omega = \Omega_e$	$1.5\ 10^{12}$	$9.5\ 10^{10}$
$\lambda[cm]$	**0.1**	**2**
Ω	$1.08\ 10^{6}$	$1.08\ 10^{6}$
y_0	$2\ 10^{-5}$	$5\ 10^{-3}$
$q\ \ (N_0 = 10^6\ [cm^{-3}])$	1.510^{-9}	3.510^{-7}
$\Delta W/W_0\quad (L = 10\ m)$	$\sim \mathbf{10^{-12}}$	$\sim \mathbf{10^{-8}}$

Conclusions

- formula describing decay of the synchrotron radiation wave in magnetized plasma of very low, but finite temperature plasma due to Cherenkov interaction with resonance electorns at frequencies near cyclotron resonance
  - polarization of the waves treated consistently
- estimated absorption of synchr. radiation in the electron cloud (only for Gaussian distr.)
  - it occurs away from the median plane
  - it is weaker than in the hypotetic case of e^- beam traversing e^- cloud
  - negiligible for LHC parameters