EFFECT OF BUNCH LENGTH, CHROMATICITY, AND LINEAR COUPLING ON THE TRANSVERSE MODE-COUPLING INSTABILITY DUE TO E$^-$CLOUD

E. Metral

- **Introduction and motivation**
- **Model used for the classical TMC instability**
  - **Bunch length**
  - **Chromaticity**
  - **Linear coupling**
- **Application to the CERN SPS beam for LHC**
- **What about the PS?**
- **Conclusion**

INTRODUCTION AND MOTIVATION

- **A vertical single-bunch instability is observed in the SPS with rise-times faster than the synchrotron period**

- **The e$^-$-cloud induced impedance can be approximated by a broad-band impedance, whose shunt impedance and resonance frequency depend on the**
  - **Bunch length**
  - **Bunch intensity**
  - **…**

- **The chromaticity helps to increase the intensity threshold**
- **Which chromaticity is predicted to stabilize the nominal beam?**
- **Can we increase the intensity threshold by**
  - **Increasing or decreasing the bunch length?**
  - **Using linear coupling?**

MODEL USED FOR THE CLASSICAL TMC INSTABILITY (1/8)

- **Formula used for the 1D TMC intensity threshold when** $2\, f_r\, \tau_b \geq 1$

$$N_b \leq \frac{4\pi^3\, f_s\, Q_y\, E\, \tau_b^2}{e\, c} \times \frac{f_r}{\left| Z_y \right|} \times \left(1 + \frac{f_{\xi_y}}{f_r} \right)$$

or

$$N_b \leq \frac{8\pi\, Q_y \left| \eta \right| \varepsilon_l}{e\, \beta^2 c} \times \frac{f_r}{\left| Z_y \right|} \times \left(1 + \frac{f_{\xi_y}}{f_r} \right)$$

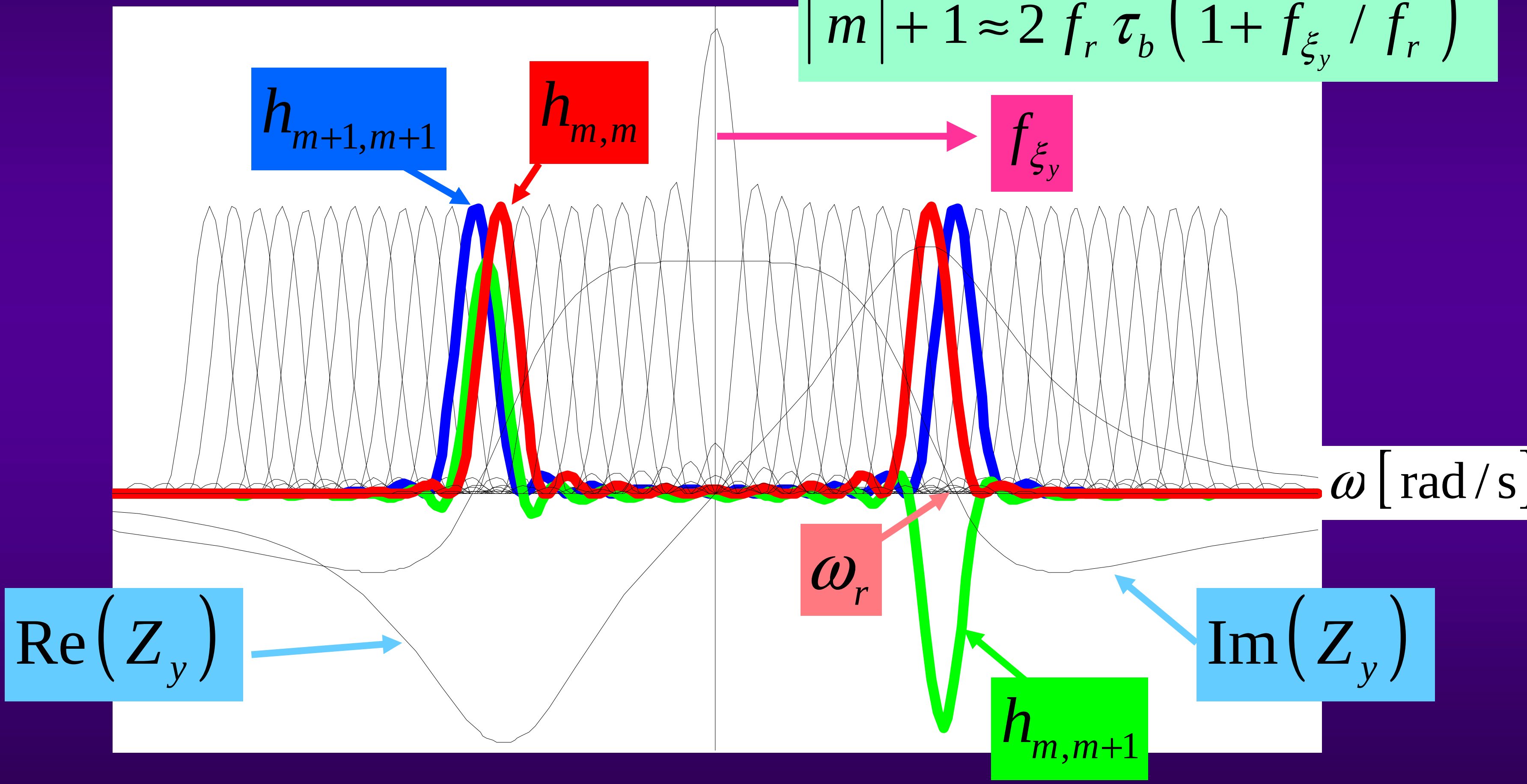
⇒ **It is the same formula as from**

- **Coasting-beam approach with peak values of bunch current and momentum spread**
- **Ruth and Wang fast blow-up theory**
- **Kernel et al. post-head-tail formalism**
- **Zotter theory for 0 chromaticity**

MODEL USED FOR THE CLASSICAL TMC INSTABILITY (2/8)

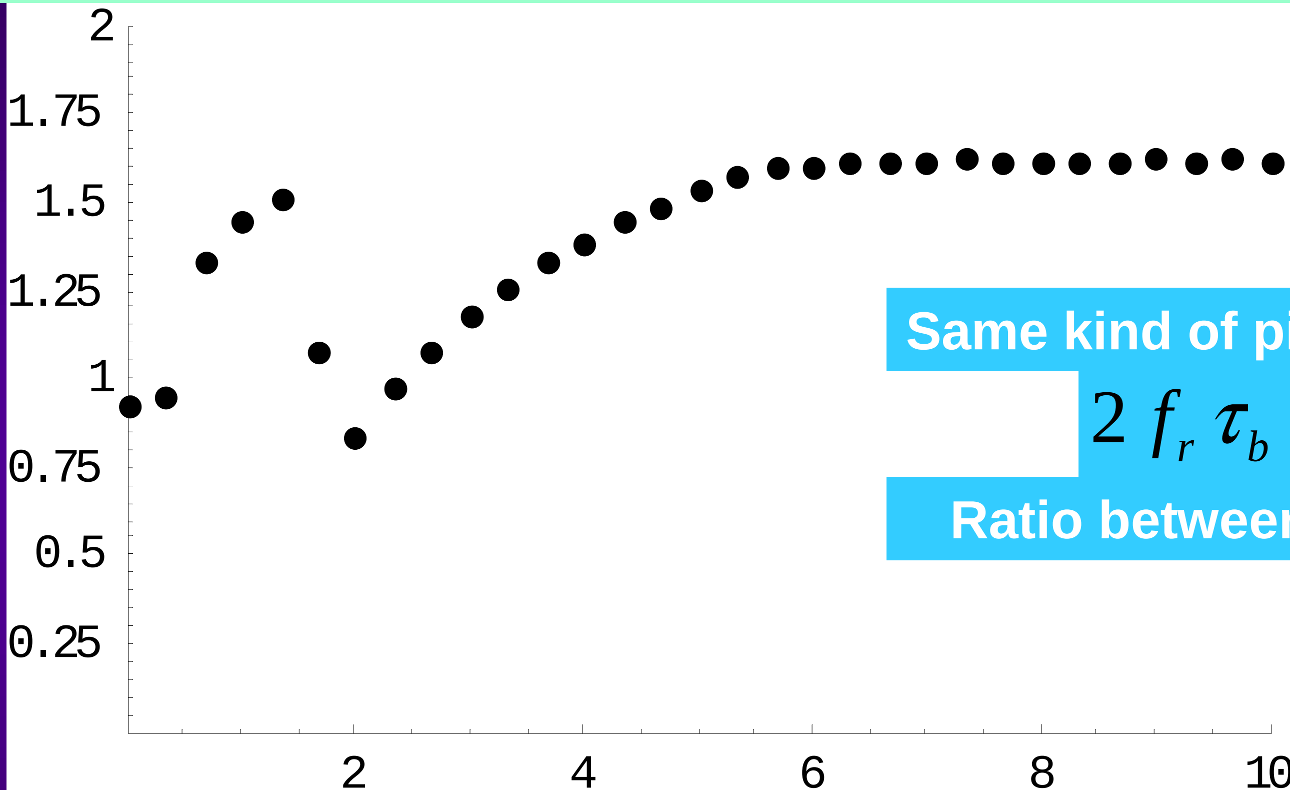
- **Consider 1st a very long bunch** $2\, f_r\, \tau_b >> 1$

$$\left| m \right| + 1 \approx 2\, f_r\, \tau_b \left(1 + f_{\xi_y} / f_r \right)$$



MODEL USED FOR THE CLASSICAL TMC INSTABILITY (3/8)

Ratio between simulated and theoretical intensity thresholds



Same kind of pictures for

$$2\, f_r\, \tau_b \geq 1$$

Ratio between 1 and 2

$f_{\xi_y} \,/\, f_r$

- **The same stability criterion is obtained for** $2\, f_r\, \tau_b \geq 1$

MODEL USED FOR THE CLASSICAL TMC INSTABILITY (4/8)

Remark : Using the same method, the stability criterion for Longitudinal Mode-Coupling instability can be derived, taking into account the potential-well distortion due to both Space-Charge and Broad-Band impedances

Below or above transition

It is $\left| Z_l \,/\, p \right|$ in the Keil-Schnell-Boussard criterion

$$\frac{\left| Z_l^{BB} / p \right|}{1.2} \times \left[1 \pm \frac{3}{4} \left(\frac{\left| Z_l^{SC} / p \right|}{\left| Z_l^{BB} / p \right|} - 1 \right) \right]^{1/4} \leq \frac{(E/e)\, \beta^2 \left| \eta \right|}{I_p} \times \left(\frac{\Delta p}{p_0} \right)^2_{\mathrm{FWHH}}$$

MODEL USED FOR THE CLASSICAL TMC INSTABILITY (5/8)

- **Formula used for the 1D TMC intensity threshold when** $2\, f_r\, \tau_b \leq 1$

$$N_b^{th,short}\left(\xi_y = 0\right) = \frac{N_b^{th,long}\left(\xi_y = 0\right)}{\left(2\, f_r\, \tau_b\right)^3} \times \frac{1}{2}\left[1 + \left(2\, f_r\, \tau_b\right)^2\right]$$

  - **It has been computed by Zotter**
  - **The ratio** $\left(2\, f_r\, \tau_b\right)^3$ **is also obtained between the Beam Break-Up "rise-times" (1 e-folding time) which can be derived from Brandt and Gareyte formula for long bunches (which is derived from Yokoya's formalism for cumulative Beam Break-Up) and from Chao et al. formula for short bunches**
  - **The threshold increases "slowly" with chromaticity**

MODEL USED FOR THE CLASSICAL TMC INSTABILITY (6/8)

- **Formula used for the 2D TMC intensity threshold**

Linear coupling term

$$
\begin{vmatrix}
\omega_c - \omega_{x,m} & -\Delta\omega^{x}_{m,m+1} & -\dfrac{\underline{\hat{K}}_0\left(l\right) R^2\,\Omega_0^2}{2\,\omega_{x0}} & 0 \\
-\Delta\omega^{x}_{m+1,m} & \omega_c - \omega_{x,m+1} & 0 & -\dfrac{\underline{\hat{K}}_0\left(l\right) R^2\,\Omega_0^2}{2\,\omega_{x0}} \\
-\dfrac{\underline{\hat{K}}_0\left(-l\right) R^2\,\Omega_0^2}{2\,\omega_{y0}} & 0 & \omega_c - \omega_{y,m} & -\Delta\omega^{y}_{m,m+1} \\
0 & -\dfrac{\underline{\hat{K}}_0\left(-l\right) R^2\,\Omega_0^2}{2\,\omega_{y0}} & -\Delta\omega^{y}_{m+1,m} & \omega_c - \omega_{y,m+1}
\end{vmatrix} = 0
$$

MODEL USED FOR THE CLASSICAL TMC INSTABILITY (7/8)

⇒ 4th order equation, which can be solved on the resonance

$$\omega_{x0} + \frac{1}{2}\left(\Delta\omega^{x}_{m,m} + \Delta\omega^{x}_{m+1,m+1}\right) = \omega_{y0} + l\,\Omega_0 + \frac{1}{2}\left(\Delta\omega^{y}_{m,m} + \Delta\omega^{y}_{m+1,m+1}\right)$$

⇒ Necessary condition for stability

$$\left|\Delta\omega^{x}_{m,m+1} + \Delta\omega^{y}_{m,m+1}\right| \leq \frac{1}{2}\left|2\,\omega_s + \Delta\omega^{x}_{m+1,m+1} + \Delta\omega^{y}_{m+1,m+1} - \Delta\omega^{x}_{m,m} - \Delta\omega^{y}_{m,m}\right|$$

Stability criterion

$$\left|\underline{\hat{K}}_0\left(l\right)\right| \geq \frac{2\sqrt{\omega_{x0}\,\omega_{y0}}}{R^2\,\Omega_0^2}\left[\frac{1}{2}\left(\Delta\omega^{x}_{m,m} - \omega_s - \Delta\omega^{x}_{m+1,m+1}\right) \mp \Delta\omega^{x}_{m,m+1}\right]^{1/2}$$

$$\left[\frac{1}{2}\left(\Delta\omega^{y}_{m,m} - \omega_s - \Delta\omega^{y}_{m+1,m+1}\right) \pm \Delta\omega^{y}_{m,m+1}\right]^{1/2}$$

MODEL USED FOR THE CLASSICAL TMC INSTABILITY (8/8)

- **Consider the case** $\xi_x = \xi_y$, $Q_x \approx Q_y$, **and** $Z_y = \alpha \, Z_x$ **(**$\alpha \geqslant 1$**)**

  ⇒ **Necessary condition for stability**

$$\left| \Delta\omega^{y}_{m,m+1} \right| \le \frac{1}{2} \left| \omega'_s + \Delta\omega^{y}_{m+1,m+1} - \Delta\omega^{y}_{m,m} \right|$$

It is the 1D stability criterion with

$$\omega_s \quad \Rightarrow \quad \omega'_s = \frac{2\,\alpha}{\alpha + 1}\,\omega_s$$

A factor 2 is gained on the threshold intensity when $\alpha \gg 1$

APPLICATION TO THE CERN SPS BEAM FOR LHC (1/5)

- **Broad-band e⁻-cloud induced impedance**
  - **Computed for SPS with**

cf. K. Ohmi et al., "Study of the Fast Head-Tail Instability Caused by Electron Cloud"

$$N_{b0} = 7.5\times10^{10}\ \mathrm{p/b} \qquad \rho_{c0} = 10^{12}\ e^- / \mathrm{m}^3$$

$$\sigma_{x0} = 5\ \mathrm{mm} \qquad \sigma_{y0} = 3\ \mathrm{mm} \qquad \sigma_{z0} = 30\ \mathrm{cm}$$

  - **Results**

$$\left| Z_{y0} \right| = 20\ \mathrm{M\Omega/m} \qquad f_{r0} = 220\ \mathrm{MHz} \qquad Q_{r0} = 1$$

$$\left| Z_y \right| = \left| Z_{y0} \right| \times \frac{\sigma_z}{\sigma_{z0}} \times \frac{\sigma_{y0}\left(\sigma_{x0}+\sigma_{y0}\right)}{\sigma_y\left(\sigma_x+\sigma_y\right)}$$

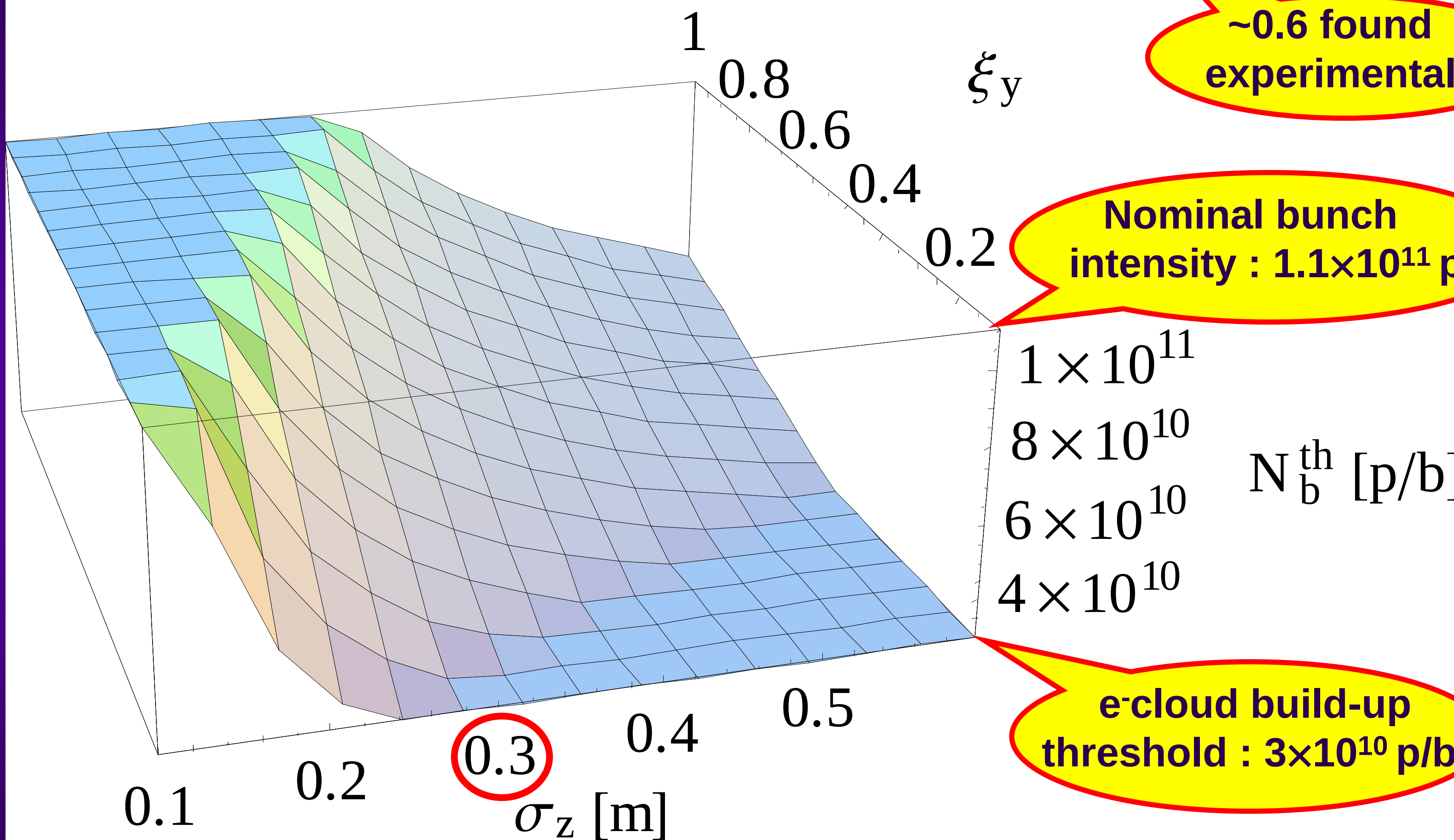
$$f_r = f_{r0} \times \sqrt{\frac{\sigma_{z0}}{\sigma_z}} \times \sqrt{\frac{N_b}{N_{b0}}} \times \sqrt{\frac{\sigma_{y0}\left(\sigma_{x0}+\sigma_{y0}\right)}{\sigma_y\left(\sigma_x+\sigma_y\right)}}$$

APPLICATION TO THE CERN SPS BEAM FOR LHC (2/5)

1D case

$N_b^{th}\left(0.3, 0.44\right) = 6\times10^{10}$ p / b

~0.6 found experimentally



Nominal bunch intensity : 1.1×10^{11} p/b

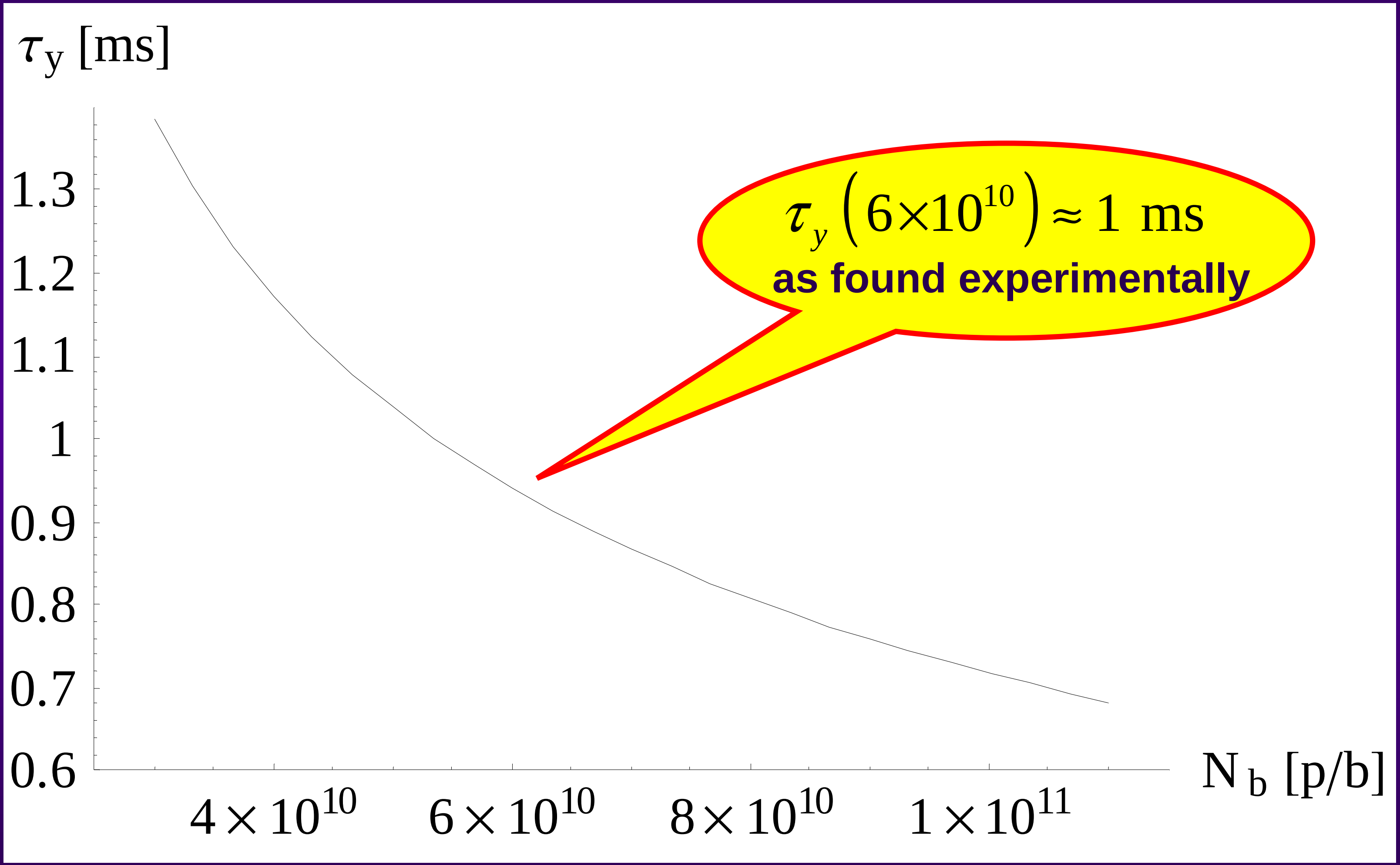
e-cloud build-up threshold : 3×10^{10} p/b

$N_b^{th}\left(0.3, 0\right) = 4.3\times10^{9}$ p / b

$N_b^{th}\left(0.3, 0.88\right) = 1.1\times10^{11}$ p / b

APPLICATION TO THE CERN SPS BEAM FOR LHC (3/5)

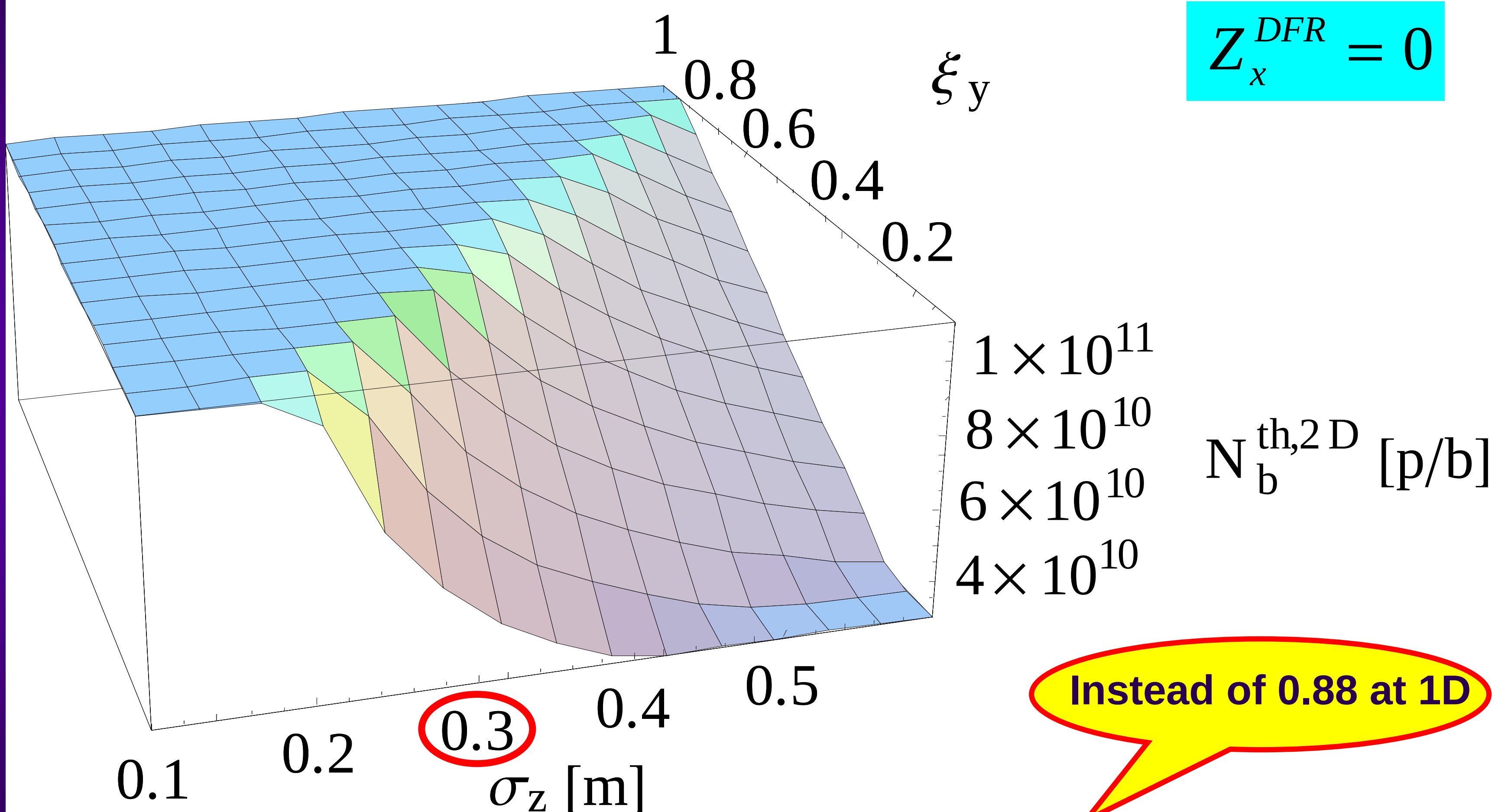
Rise-Time $\xi_y = 0$



APPLICATION TO THE CERN SPS BEAM FOR LHC (4/5)

2D case, considering only Dipole Field Regions

$Z_x^{DFR} = 0$

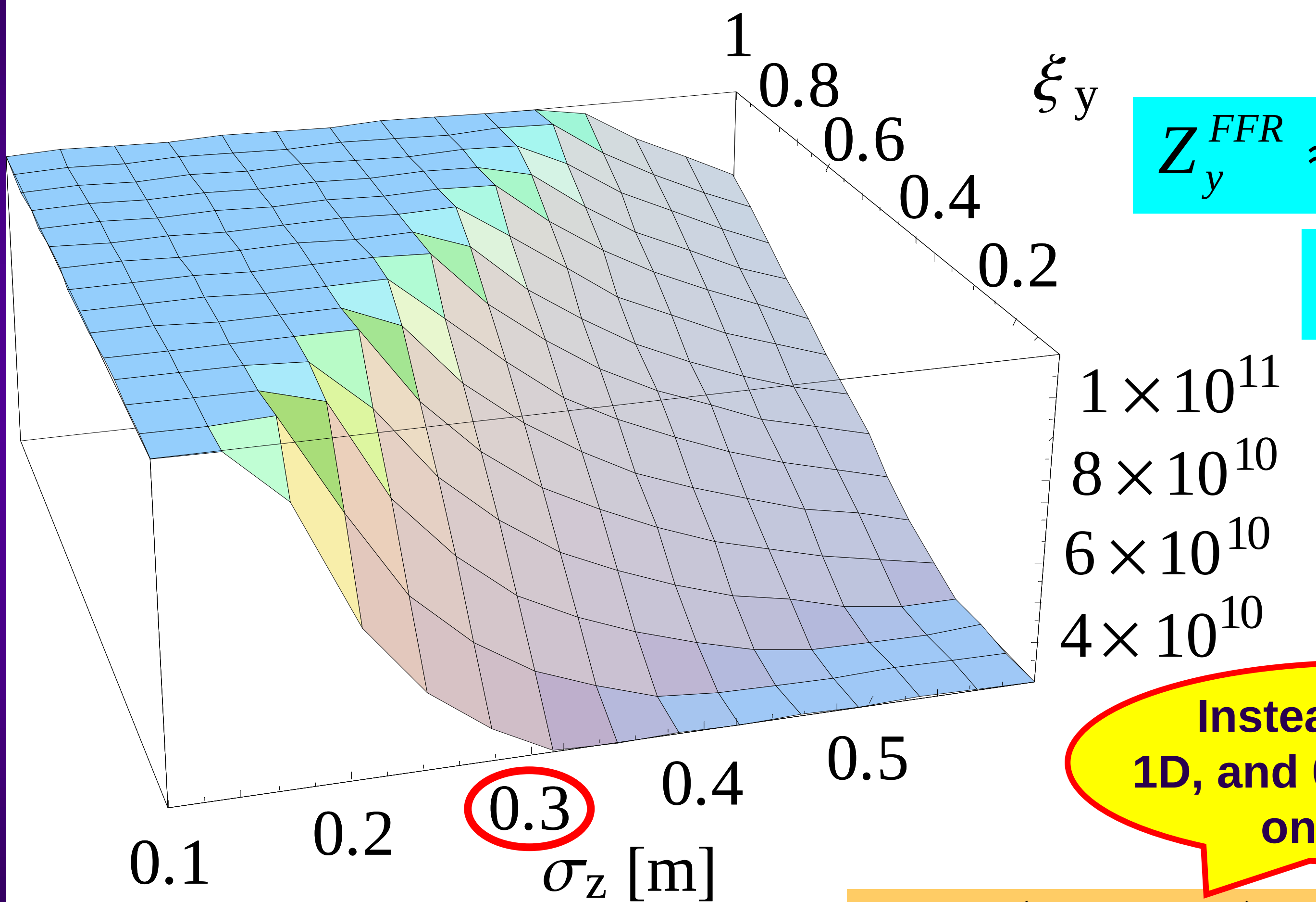


Instead of 0.88 at 1D

$N_b^{th,2D}\left(0.3, 0.34\right) = 1.1 \times 10^{11}\ \mathrm{p/b}$

APPLICATION TO THE CERN SPS BEAM FOR LHC (5/5)

2D case, considering Dipole Field Regions (2/3) and Field-Free Regions (1/3)



$$Z_y^{FFR} \approx Z_x^{FFR} \approx Z_y^{DFR}$$

$$Z_x^{DFR} = 0$$

Instead of 0.88 at 1D, and 0.34 a 2D with only DFR

$$N_b^{th,2D}\left(0.3, 0.52\right) = 1.1 \times 10^{11} \text{ p / b}$$

WHAT ABOUT THE PS (1/2)

- Single-bunch instability observed in 2001 in the PS on a modified PS beam for LHC (bunch length of ~10 ns during ~100 ms before extraction
- Rise-times of few ms

$$T_s = 1.4 \text{ ms}$$

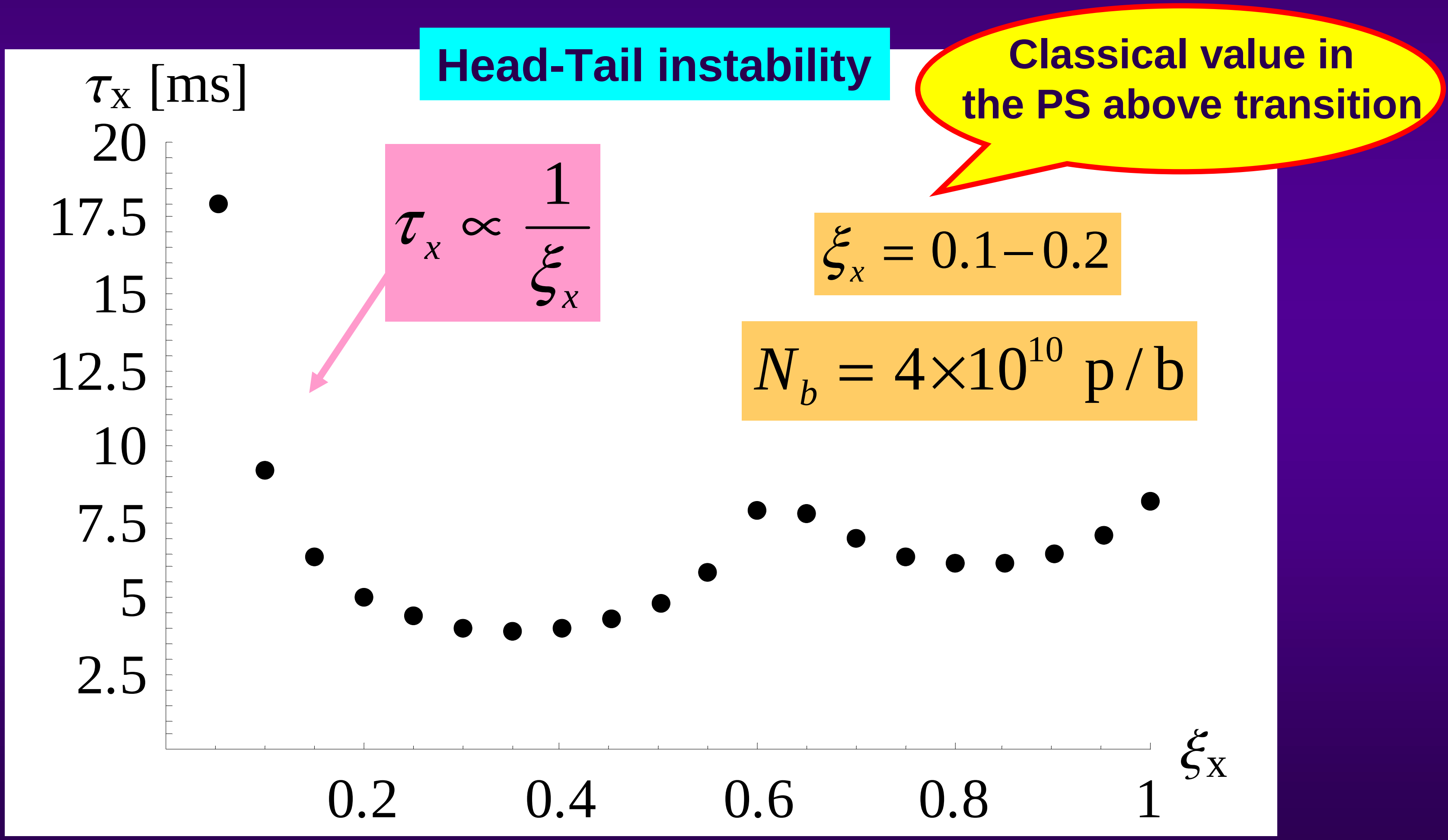
- In the horizontal plane, whereas it is on the vertical plane for the SPS
  - In the PS : 90% of DFR and 10% of FFR
  - In the SPS : 2/3 of DFR and 1/3 of FFR
- Combined-function magnets are used in the PS, whereas separate-function magnets are used in the SPS

It may explain why the instability is observed in the horizontal plane… cf. G. Rumolo's talk on the PS electron cloud effects

WHAT ABOUT THE PS (2/2)

VERY PRELIMINARY RESULT with the horizontal wake-field computed by simulation by G. Rumolo in the PS combined-function magnet

Head-Tail instability



CONCLUSION (1/2)

- The formula used for the classical TMC intensity threshold is the same as from
  - Coasting-beam approach with peak values of bunch current and momentum spread
  - Ruth and Wang fast blow-up theory
  - Kernel et al. post-head-tail formalism

- It is applied to the SPS with the vertical broad-band e^-cloud induced impedance. Higher intensity thresholds can be reach by
  - Increasing the chromaticity
  - Decreasing the bunch length
  - Using linear coupling

- The predicted SPS rise-time and stabilizing effect of chromaticity are in quantitative agreement with the observations (done until $\sim 6 \times 10^{10}$ p/b)

CONCLUSION (2/2)

- **In the PS, the combined-function magnets may explain why a horizontal instability is observed (VERY PRELIMINARY RESULTS ⇒ To be continued)**

- **The PS transverse impedances computed by simulation could explain**
  - **The plane of the instability**
  - **The observed rise-times of few ms (several synchrotron periods ⇒ Head-Tail regime)**
  - **No stabilizing effect by increasing the chromaticity**

⇒ To be confirmed…

ACKNOWLEDGEMENTS

Many thanks to G. Arduini, M. Giovannozzi, G. Rumolo and F. Zimmermann for very helpful discussions