

EFFECT OF BUNCH LENGTH, CHROMATICITY, AND LINEAR COUPLING ON THE TRANSVERSE MODE-COUPLING INSTABILITY DUE TO THE ELECTRON CLOUD

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Abstract

The influence of bunch length, chromaticity, and linear coupling on the transverse mode-coupling intensity threshold is discussed for the case of a bunch interacting with a broad-band resonator impedance. Two regimes are possible according to whether the total bunch length is above or below a critical value, which is about the inverse of twice the resonance frequency. If the bunch length is greater than this value, the intensity threshold in the absence of linear coupling can be approximated by the coasting-beam value multiplied by the bunching factor. Hence, it is proportional to the bunch length, and increases linearly with the ratio between the chromatic and resonance frequencies. If the bunch length is smaller than the critical value, the intensity threshold is inversely proportional to the square of the bunch length, but it still increases slowly with chromaticity. In the presence of linear coupling, the intensity threshold can be increased up to a factor two when the second transverse plane has a negligible impedance. This formalism is applied to the broad-band electron-cloud induced impedance, to evaluate the effect of bunch length, chromaticity and linear coupling on the intensity threshold of the CERN SPS beam for LHC.

1 INTRODUCTION

A vertical single-bunch instability due to the electron cloud is observed in the SPS, with rise-times faster than the synchrotron period [1,2]. The electron-cloud induced impedance has been approximated by a broad-band impedance ($Q_r \approx 1$), whose shunt impedance and resonance frequency depend on bunch length and intensity [3]. Furthermore, it has been found experimentally that increasing the chromaticity helps to increase the intensity threshold.

The purpose of this paper is to compare these observations with theoretical predictions, by evaluating the effect of bunch length and chromaticity on the intensity threshold of the CERN SPS beam for LHC. Another parameter, which could be used to increase the intensity threshold is proposed: this is linear coupling

between the transverse planes.

The model used for the classical one- and two-dimensional Transverse Mode-Coupling (TMC) instability is described and discussed in Section 2. This formalism is then applied to the CERN SPS beam for LHC in Section 3.

2 THEORY

2.1 One-Dimensional

Considering the case where two adjacent head-tail modes (m and $m+1$) undergo a coupled motion, the stability of a high-intensity single-bunch beam can be discussed using the following determinant, e.g. for the vertical plane, [4]

$$\begin{vmatrix} \omega_c - \omega_{y,m} & -\Delta\omega_{m,m+1}^y \\ -\Delta\omega_{m+1,m}^y & \omega_c - \omega_{y,m+1} \end{vmatrix} = 0, \quad (1)$$

with

$$\Delta\omega_{m,n}^y = (|m|+1)^{-1} \frac{j e \beta I_b}{2 m_0 \gamma Q_{y0} \Omega_0 L} (Z_y^{eff})_{m,n}, \quad (2)$$

$$(Z_y^{eff})_{m,n} = \frac{\sum_{k=-\infty}^{k=+\infty} Z_y(\omega_k^y) h_{m,n}(\omega_k^y - \omega_{\xi_y})}{\sum_{k=-\infty}^{k=+\infty} h_{m,m}(\omega_k^y - \omega_{\xi_y})}, \quad (3)$$

$$h_{m,n}(\omega) = \frac{\tau_b^2}{\pi^4} (|m|+1) \times (|n|+1) \times F_m^n \times \left\{ (\omega \tau_b / \pi)^2 - (|m|+1)^2 \right\}^{-1} \times \left\{ (\omega \tau_b / \pi)^2 - (|n|+1)^2 \right\}^{-1}, \quad (4)$$

$$F_{m \text{ even}}^n = (-1)^{(|m|+|n|)/2} \times \cos^2[\omega \tau_b / 2], \quad (5)$$

$$F_{m \text{ odd}}^n = \frac{(-1)^{(|m|+|n|+3)/2}}{2j} \times \sin[\omega \tau_b], \quad (6)$$

$$F_{m\text{ odd}}^{n\text{ even}} = \frac{(-1)^{(|m|+|n|+1)/2}}{2j} \times \sin[\omega\tau_b], \quad (7)$$

$$F_{m\text{ odd}}^{n\text{ odd}} = (-1)^{(|m|+|n|+2)/2} \times \sin^2[\omega\tau_b/2]. \quad (8)$$

Here, ω_c is the coherent angular frequency to be determined, $\omega_{y,m} = \omega_{y0} + m\omega_s + \Delta\omega_{m,m}^y$, with $\omega_{y0} = Q_{y0}\Omega_0$ the unperturbed betatron angular frequency with Q_{y0} the unperturbed tune and $\Omega_0 = 2\pi f_0$ the revolution angular frequency, $m = \dots, -1, 0, 1, \dots$ is the head-tail mode number, $\omega_s = 2\pi f_s$ is the synchrotron angular frequency (T_s is the synchrotron period), $j = \sqrt{-1}$ is the imaginary unit, e is the elementary charge, β and γ are the relativistic velocity and mass factors, $I_b = N_b e f_0$ is the current in one bunch with N_b the number of protons in the bunch, m_0 is the proton rest mass, $L = \beta c \tau_b$ is the full (4σ) bunch length (in metres) with c the speed of light and τ_b the total bunch length (in seconds), Z_y is the coupling impedance, $\omega_k^y = (k + Q_{y0})\Omega_0 + m\omega_s$ with $-\infty \leq k \leq +\infty$, $\omega_{\xi_y} = 2\pi f_{\xi_y} = (\xi_y/\eta) Q_{y0} \Omega_0$ is the chromatic angular frequency, with $\xi_y = (\Delta Q_y / \Delta p) (p_0 / Q_{y0})$ and $\eta = \gamma_{tr}^{-2} - \gamma^{-2} = (\Delta T / T_0) / (\Delta p / p_0)$ the chromaticity and slippage factor, where p is the momentum and T the revolution period of a particle, and $h_{m,n}$ describes the cross-power densities of the m th and n th line-density modes. As can be seen from Eqs. (4-8), $h_{m,m+1}(\omega) = -h_{m+1,m}(\omega)$, which yields

$$\Delta\omega_{m+1,m}^y = -k_m^2 \Delta\omega_{m,m+1}^y, \quad (9)$$

with

$$k_m = \sqrt{\frac{|m|+1}{|m+1|+1}}. \quad (10)$$

This parameter is often approximated by one [4]. Considering the case of a driving broad-band resonator, the coupling impedance is given by

$$Z_y(\omega) = \frac{\omega_r}{\omega} R_r / \left[1 - jQ_r \left(\frac{\omega_r}{\omega} - \frac{\omega}{\omega_r} \right) \right], \quad (11)$$

where $\omega_r = 2\pi f_r$ is the resonance angular frequency, Q_r the quality factor and R_r the shunt impedance. Equation (1) leads to the following solutions for ω_c

$$\omega_c^\pm = \frac{1}{2} \times \left[2\omega_{y0} + (2m+1)\omega_s + \Delta\omega_{m,m}^y + \Delta\omega_{m+1,m+1}^y \right] \pm \frac{1}{2} \sqrt{(\omega_s + \Delta\omega_{m+1,m+1}^y - \Delta\omega_{m,m}^y)^2 - 4k_m^2 (\Delta\omega_{m,m+1}^y)^2}. \quad (12)$$

In the following, only the real parts of the complex betatron frequency shifts are considered (see Section 2.2 for a discussion on the model used). We thus write

$$\Delta\omega_{m,m}^y = a_0 I_b, \quad (13)$$

$$\Delta\omega_{m+1,m+1}^y = b_0 I_b, \quad (14)$$

$$\Delta\omega_{m,m+1}^y = c_0 I_b, \quad (15)$$

$$I_{b,th1} = \frac{\omega_s}{a_0 - b_0 + 2k_m |c_0|}, \quad (16)$$

$$I_{b,th2} = \frac{\omega_s}{a_0 - b_0 - 2k_m |c_0|}. \quad (17)$$

If $I_{b,th2} > 0$, then $I_{b,th2} > I_{b,th1}$. The beam is stable from zero intensity to $I_{b,th1}$. Then it is unstable between $I_{b,th1}$ and $I_{b,th2}$ (mode-coupling at $I_{b,th1}$). Finally, it is stable again above $I_{b,th2}$ (mode-decoupling at $I_{b,th2}$). This case is depicted in Fig. 1.

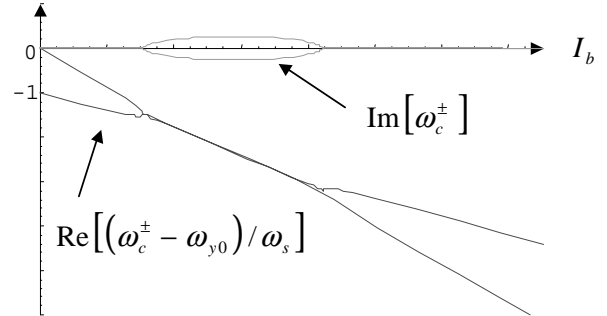


FIGURE 1. Mode-coupling and -decoupling: the upper traces describe the imaginary parts of the coherent betatron frequencies ω_c^\pm , and the lower traces the real parts.

This corresponds to the case of a long bunch, whose spectra of modes 0 and -1 peak at low frequencies. Both modes couple to the inductive part of the coupling impedance, and therefore are shifted in the same direction. Moreover, their coupling to the resistive part of the coupling impedance is weak. As a consequence, when the two modes merge, they cannot develop a strong instability and are pulled apart as intensity increases. Modes of higher order can couple, but higher-order modes are more difficult to drive than lower-order ones [5].

Therefore, in our model one will always consider mode-coupling between the two most critical head-tail modes (m and $m+1$) overlapping the peak of the negative resistive impedance. In this case there will never be

mode-decoupling ($I_{b,th2} < 0$), and the threshold for mode-coupling is obtained at the intensity $I_{b,th1}$ (see Fig. 2).

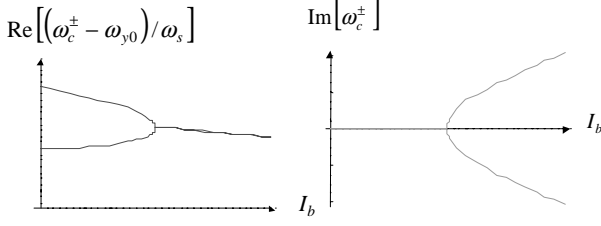


FIGURE 2. Real (left) and imaginary (right) parts of the coherent betatron frequencies ω_c^\pm .

Below the intensity threshold $I_{b,th1}$, the real and imaginary parts of the coherent frequencies are given by

$$\text{Re}(\omega_c^\pm) = \omega_{y0} + (m+1/2)\omega_s + I_b(a_0 + b_0)/2 \quad (18)$$

$$\pm \frac{1}{2} \sqrt{[\omega_s + (b_0 - a_0)I_b]^2 - 4k_m^2 c_0^2 I_b^2},$$

$$\text{Im}(\omega_c^\pm) = 0. \quad (19)$$

Above the intensity threshold $I_{b,th1}$, the real and imaginary parts of the coherent frequencies are given by

$$\text{Re}(\omega_c^\pm) = \omega_{y0} + (m+1/2)\omega_s + I_b(a_0 + b_0)/2, \quad (20)$$

$$\text{Im}(\omega_c^\pm) = \pm \frac{1}{2} \sqrt{4k_m^2 c_0^2 I_b^2 - [\omega_s + (b_0 - a_0)I_b]^2}. \quad (21)$$

The instability rise-times are given by

$$\tau_\pm = \frac{-1}{\text{Im}(\omega_c^\pm)}. \quad (22)$$

The rise-time of the unstable mode can be re-written

$$\tau_- = T_s \times \frac{1}{\pi \sqrt{(\alpha-1)(\alpha q+1)}}, \quad (23)$$

where q and α are given by

$$q = \frac{2k_m |c_0| + b_0 - a_0}{2k_m |c_0| - b_0 + a_0}, \quad \alpha = \frac{I_b}{I_{b,th1}}. \quad (24)$$

The parameter $q \in [0,1]$: it is equal to one for long bunches and zero for short bunches. The two curves describing the two extreme cases for the rise-time given

by Eq. (23) are shown in Fig. 3. It can thus be seen from Fig. 3, that the same (well-known) result is obtained in all cases: just above threshold, the instability rise-time is given by the synchrotron period.

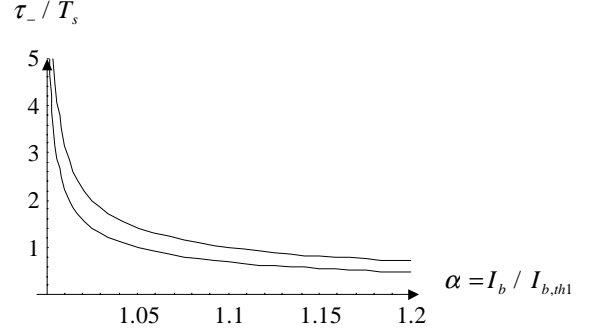


FIGURE 3. Instability rise-time normalised to the synchrotron period vs. $\alpha = I_b / I_{b,th1}$, for $q=0$ (upper trace) and $q=1$ (lower trace).

Investigate now the effect of bunch length and chromaticity on the intensity threshold. Two regimes are possible according to whether the total bunch length is above or below a critical value, which is about the inverse of twice the resonance frequency (see Fig. 4). This corresponds to the time when the wake-field becomes negative (see Fig. 5). In the frequency domain, this case corresponds to the second picture of Fig. 6.

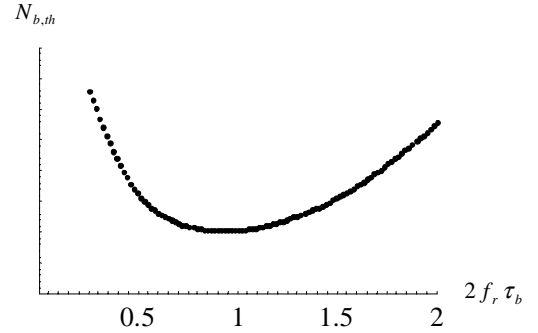


FIGURE 4. Intensity threshold near $2f_r \tau_b = 1$, by solving numerically Eq. (1) for modes 0 and -1.

Transverse wake-field

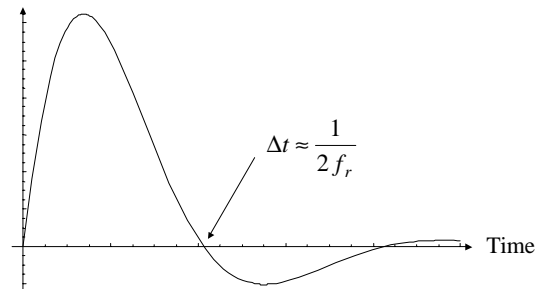


FIGURE 5. Transverse wake-field vs. time.

If $\tau_b \geq 0.5/f_r$, it is the “long-bunch” regime (see Fig. 6).

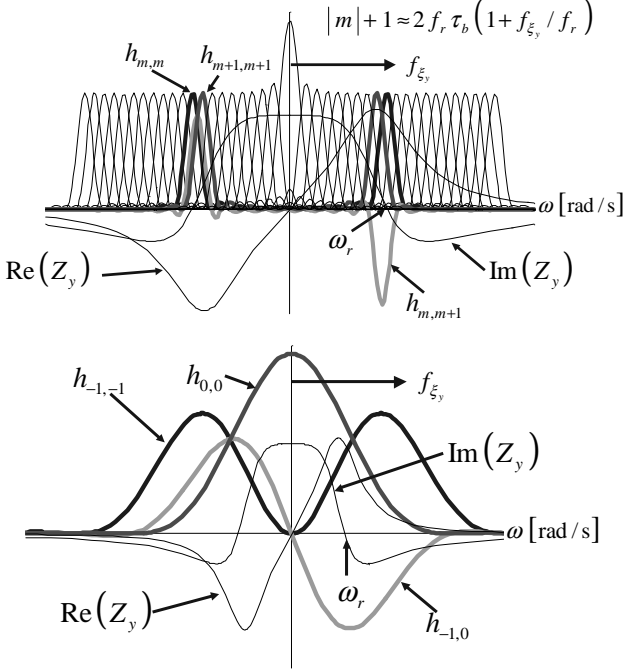


FIGURE 6. Power spectra for a long ($\tau_b \gg 0.5/f_r$) and short ($\tau_b = 0.5/f_r$) bunch, in the “long-bunch” regime, and real and imaginary parts of the driving broad-band impedance.

The intensity threshold can be approximated by [4]

$$N_{b,th} = \frac{4\pi^3 f_s Q_{y0} E \tau_b^2}{ec} \times \frac{f_r}{|Z_y|} \times \left(1 + \frac{f_{\xi_y}}{f_r} \right), \quad (25)$$

which can be re-written,

$$N_{b,th} = \frac{8\pi Q_{y0} |\eta| \varepsilon_l}{e\beta^2 c} \times \frac{f_r}{|Z_y|} \times \left(1 + \frac{f_{\xi_y}}{f_r} \right), \quad (26)$$

using $f_s = |\eta| \times (\Delta p/p_0)_{\max} / (\pi\tau_b)$ and approximating the longitudinal emittance (at 2σ , in eV.s), by an elliptic area in the longitudinal phase space, which gives $\varepsilon_l = \beta^2 E \tau_b (\Delta p/p_0)_{\max} \pi/2$, where E is the total beam energy, and $|Z_y|$ is the peak value of the resonator. Formula (26) is the same as from (i) the coasting-beam approach using the peak values of bunch current and momentum spread [6], (ii) Ruth and Wang fast blow-up theory [7], (iii) Kernel et al. post-head-tail formalism [8], and (iv) Zotter theory for zero

chromaticity [9]. In fact, if one computes the ratio between the intensity threshold obtained by simulation, solving numerically Eq. (1) for the two most critical modes given by

$$|m| + 1 \approx 2 f_r \tau_b \left(1 + \frac{f_{\xi_y}}{f_r} \right), \quad (27)$$

and the intensity threshold given by Eq. (26), the same kind of pictures as in Fig. 7 are obtained. The ratio is always between ~ 1 and ~ 2 , as can be easily deduced from the first picture of Fig. 6 for a very long bunch. It is approximated by one [4].

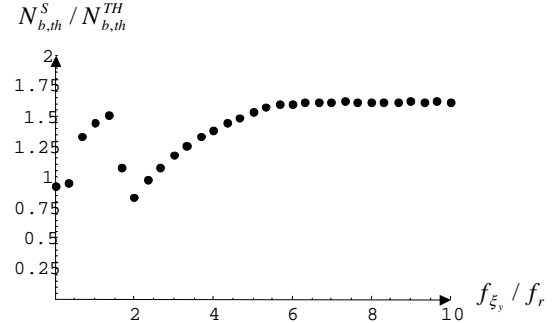


FIGURE 7. Ratio between the intensity threshold $N_{b,th}^S$ computed numerically from Eq. (1) with $2f_r\tau_b=10$, and the intensity threshold $N_{b,th}^{TH}$ given by Eq. (26), vs. f_{ξ_y}/f_r .

If $\tau_b < 0.5/f_r$, it is the “short-bunch” regime. The intensity threshold can be approximated by [9]

$$N_{b,th}^{short}(\xi_y = 0) = \frac{N_{b,th}(\xi_y = 0)}{(2f_r\tau_b)^3} \times \frac{1}{2} [1 + (2f_r\tau_b)^2]. \quad (28)$$

The second term is a small term varying between $1/2$ (when $\tau_b \ll 0.5/f_r$) and 1 (when $\tau_b = 0.5/f_r$). Note that the factor $(2f_r\tau_b)^3$ is also obtained between the beam break-up “rise-times” (one e -folding time), which can be derived from Brandt and Gareyte formula [10] for long bunches (which is derived from Yokoya’s formalism for cumulative beam break-up [11]) and from Chao et al. for short bunches [12]. The intensity threshold increases “slowly” with chromaticity, as the bunch spectrum for mode 0 extends well above the resonance frequency.

2.2 Discussion on the Model Used

The model used here for the classical TMC instability is based on the mode-coupling between the two most critical head-tail modes (m and $m+1$) overlapping the

peak of the negative resistive impedance. For zero chromaticity, the tune shifts are real. There is no Head-Tail (HT) instability, and above a threshold intensity, a TMC instability develops, with an instability rise-time faster than the synchrotron period. When the chromatic frequency is shifted positively (this is the stability criterion for the head-tail mode $m=0$), the simple model where the two regimes (HT and TMC) are treated separately is used here. Below the threshold intensity, the standing-wave patterns (head-tail modes) are treated independently. Above the threshold intensity, the wake fields couple the head-tail modes together and a travelling-wave pattern is created along the bunch. This is the TMC instability. In this paper, only the TMC intensity threshold is looked at, i.e. only the real parts of the complex coherent tune shifts are considered. Other people [13] use directly the complex tune shifts, and solve the problem numerically, thus treating both regimes at the same time. The advantage of the present formalism is that it is simple, and that it makes the link between the TMC formalism and the formula already derived when (i) the bunch length is much longer than the inverse of twice the resonance frequency, and (ii) the synchrotron frequency tends to zero (e.g. at transition), or the instability rise-time is much faster than the synchrotron period.

Note that using the same ‘‘simple’’ model, i.e. considering only the mode-coupling between the two most critical modes overlapping the peak of the negative resistance, the following stability criterion is obtained for the longitudinal mode-coupling instability, taking into account the potential-well distortion due to both space-charge and broad-band impedances, [14]

$$\frac{|Z_l^{BB}/p|}{1.2} \times \left[1 \pm \frac{3}{4} \left(\frac{|Z_l^{SC}/p|}{|Z_l^{BB}/p|} - 1 \right) \right]^{1/4} \leq \frac{(E/e) \beta^2 |\eta|}{I_p} \times \left(\frac{\Delta p}{P_0} \right)_{\text{FWHH}}^2. \quad (29)$$

Here, $I_p = 3 e N_b / (2 \tau_b)$ is the bunch peak current considering a parabolic line density, $|Z_l^{BB}(p)/p|$ and $|Z_l^{SC}(p)/p|$ are the peak values of the broad-band and space-charge longitudinal impedances, and the signs \pm correspond to the cases below and above transition respectively. It is believed that this stability criterion is more appropriate than the one of Keil-Schnell-Boussard [15] (given by Eq. (29) with the term on the left replaced by the modulus of the coupling impedance). It is known that the Keil-Schnell stability criterion [16]

for the longitudinal microwave instability of coasting beams is already a simplification, which is valid when the inductive part of the coupling impedance is smaller than the real one. When the inductive part of the coupling impedance is much greater than the real one, this stability criterion is not valid, and one has to look at the stability diagram, which is a graphical representation of the solution of the dispersion relation depicting curves of constant growth rates, and especially a threshold contour in the complex plane of the driving impedance. Equation (29) also depicts a threshold contour in the complex plane of the driving impedance.

2.3 Two-Dimensional

In the presence of linear coupling, the 2×2 determinant of Eq. (1) becomes a 4×4 determinant given by (near the coupling resonance $Q_x - Q_y = l$) [17]

$$\begin{vmatrix} \omega_c - \omega_{x,m} & -\Delta\omega_{m,m+1}^x & -\frac{\hat{K}_0(l) R^2 \Omega_0^2}{2 \omega_{x0}} & 0 \\ -\Delta\omega_{m+1,m}^x & \omega_c - \omega_{x,m+1} & 0 & -\frac{\hat{K}_0(l) R^2 \Omega_0^2}{2 \omega_{x0}} \\ -\frac{\hat{K}_0(-l) R^2 \Omega_0^2}{2 \omega_{y0}} & 0 & \omega_c - \omega_{y,m} & -\Delta\omega_{m,m+1}^y \\ 0 & -\frac{\hat{K}_0(-l) R^2 \Omega_0^2}{2 \omega_{y0}} & -\Delta\omega_{m+1,m}^y & \omega_c - \omega_{y,m+1} \end{vmatrix} = 0, \quad (30)$$

where $\omega_{x,m} = \omega_{x0} + m \omega_s + \Delta\omega_{m,m}^x$ and here $\omega_{y,m} = \omega_{y0} + l \Omega_0 + m \omega_s + \Delta\omega_{m,m}^y$, $\hat{K}_0(l)$ is the l th Fourier coefficient of the skew gradient $\underline{K}_0 = (e/p_0)(\partial B_x / \partial x)$, with B_x the horizontal magnetic field, and R is the average radius of the machine. Equation (30) leads to a fourth-order equation, which can be solved on the resonance (using here the approximation $k_m \approx 1$)

$$\begin{aligned} Q_{x0} + \frac{1}{2\Omega_0} (\Delta\omega_{m,m}^x + \Delta\omega_{m+1,m+1}^x) = \\ Q_{y0} + l + \frac{1}{2\Omega_0} (\Delta\omega_{m,m}^y + \Delta\omega_{m+1,m+1}^y). \end{aligned} \quad (31)$$

A necessary condition for stability is given by

$$\begin{aligned} |\Delta\omega_{m,m+1}^x + \Delta\omega_{m,m+1}^y| \leq \\ \frac{1}{2} |2\omega_s + \Delta\omega_{m+1,m+1}^x + \Delta\omega_{m+1,m+1}^y - \Delta\omega_{m,m}^x - \Delta\omega_{m,m}^y|. \end{aligned} \quad (32)$$

If Eq. (32) is fulfilled, then it is possible to stabilise the beam by linear coupling. Beam stability is obtained above a certain threshold for the coupling strength, whose value is given by

$$\begin{aligned}
|\hat{K}_0(l)| &\geq \frac{2\sqrt{Q_{x0}Q_{y0}}}{R^2\Omega_0} \\
&\times \left[\frac{1}{2}(\Delta\omega_{m,m}^x - \omega_s - \Delta\omega_{m+1,m+1}^x) \mp \Delta\omega_{m,m+1}^x \right]^{1/2} \\
&\times \left[\frac{1}{2}(\Delta\omega_{m,m}^y - \omega_s - \Delta\omega_{m+1,m+1}^y) \pm \Delta\omega_{m,m+1}^y \right]^{1/2}.
\end{aligned} \quad (33)$$

Consider for instance the case where $\xi_x = \xi_y$, $Q_x = Q_y$, and $Z_y = \lambda Z_x$. The necessary condition for stability of Eq. (32) becomes

$$|\Delta\omega_{m,m+1}^y| \leq \frac{1}{2} |\omega'_s + \Delta\omega_{m+1,m+1}^y - \Delta\omega_{m,m}^y|, \quad (34)$$

which is the one-dimensional vertical stability criterion with the angular synchrotron frequency ω_s replaced by

$$\omega'_s = \omega_s \times \frac{2\lambda}{\lambda+1}. \quad (35)$$

A factor 2 is thus gained on the intensity threshold when $\lambda \gg 1$, i.e. when the second transverse plane has a negligible impedance. Note that in the case $\lambda \gg 1$, the same result is obtained for different chromaticities and tunes.

3 APPLICATION TO THE CERN SPS BEAM FOR LHC

The SPS electron-cloud induced impedance has been computed in Ref. 3 for $N_{b0} = 7.5 \times 10^{10}$ p/b, considering an average density of the electron cloud of $\rho_{c0} = 10^{12}$ e⁻/m³, longitudinal and transverse rms dimensions $\sigma_{z0} = 30$ cm, $\sigma_{x0} = 5$ mm and $\sigma_{y0} = 3$ mm. The result is a broad-band impedance ($Q_r \approx 1$), with peak value $|Z_{y0}| = 20$ M Ω /m and resonance frequency $f_{r0} = 220$ MHz. Furthermore, the peak impedance and resonance frequency scale as follows

$$|Z_y| = |Z_{y0}| \times \frac{\sigma_z}{\sigma_{z0}} \times \frac{\sigma_{y0}(\sigma_{x0} + \sigma_{y0})}{\sigma_y(\sigma_x + \sigma_y)}, \quad (36)$$

$$f_r = f_{r0} \times \sqrt{\frac{\sigma_{z0}}{\sigma_z}} \times \sqrt{\frac{N_b}{N_{b0}}} \times \sqrt{\frac{\sigma_{y0}(\sigma_{x0} + \sigma_{y0})}{\sigma_y(\sigma_x + \sigma_y)}}. \quad (37)$$

Applying Eq. (26) to find the intensity threshold with the above impedance, since $\tau_b \geq 0.5/f_r$ in the cases studied, yields

$$N_{b,th} = \frac{f^2}{4\tau_b^3} \times \left(1 + \sqrt{1 + \frac{4g\xi_y\tau_b^2}{f}} \right)^2, \quad (38)$$

with

$$f = \frac{8\pi Q_{y0} |\eta| \epsilon_l f_{r0} \tau_{b0}^{3/2}}{e\beta^2 c |Z_{y0}| \sqrt{N_{b0}}} \times \sqrt{\frac{\sigma_y(\sigma_x + \sigma_y)}{\sigma_{y0}(\sigma_{x0} + \sigma_{y0})}}, \quad (39)$$

$$g = \frac{Q_{y0} f_0 \sqrt{N_{b0}}}{\eta f_{r0} \tau_{b0}^{1/2}} \times \sqrt{\frac{\sigma_y(\sigma_x + \sigma_y)}{\sigma_{y0}(\sigma_{x0} + \sigma_{y0})}}. \quad (40)$$

The plot of Eq. (38), which describes the intensity threshold vs. both chromaticity and bunch length, is shown in Fig. 8, using the numerical values given in Table 1.

Table 1: Basic parameters of the CERN SPS.

Average machine radius	R [m]	1100
Slippage factor	η	5.5×10^{-4}
Beam energy	E [GeV]	26
Nominal bunch population	N_b [p/b]	11×10^{10}
Long. emittance (2σ)	ϵ_l [eV.s]	0.35
Nominal bunch length	σ_z [cm]	30
Nominal horiz. beam size	σ_x [mm]	2.6
Nominal vert. beam size	σ_y [mm]	1.9
Vertical tune	Q_y	~ 26.7

Note that the nominal bunch length is $\sigma_z = 30$ cm, which corresponds to $\tau_b = 4$ ns. It is found that an instability is predicted once the electron-cloud build-up is predicted and observed (3×10^{10} p/b). Keeping the same bunch length, it is predicted from Fig. 8 that the nominal beam intensity should be reached for a chromaticity of 0.88. Note that for 6×10^{10} p/b, beam stability is predicted for a chromaticity of 0.44, whereas ~ 0.6 has been found experimentally. The theoretical predictions seem therefore to be in good agreement with observations. It is also seen from Fig. 8, that the intensity threshold can be raised by decreasing the bunch length. Figure 9 gives the rise-time of the instability vs. bunch intensity. It is seen that for 6×10^{10} p/b, a rise-time

of ~ 1 ms is predicted, which is in agreement with the observations.

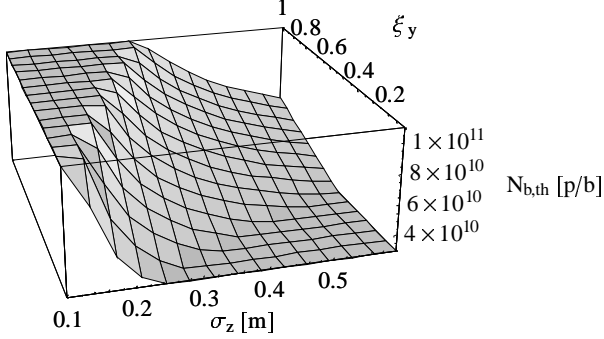


FIGURE 8. Intensity threshold of the SPS beam for LHC vs. both chromaticity and bunch length, between 3×10^{10} p/b, which is the threshold for the electron-cloud build-up, and 11×10^{10} p/b, which is the nominal intensity.

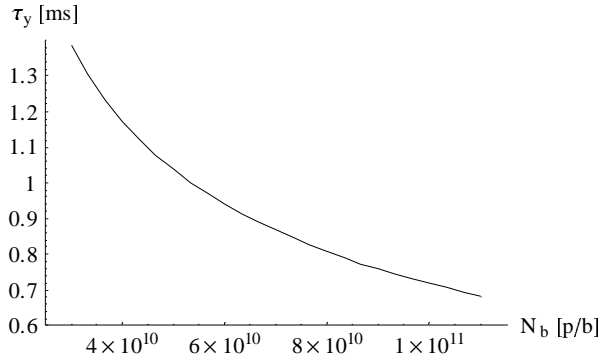


FIGURE 9. Vertical instability rise-time for zero chromaticity vs. bunch intensity.

Since 2/3 of the SPS circumference is composed of dipole-field regions, where the horizontal electron-cloud induced impedance is zero, linear coupling could be used to raise the intensity threshold. Figure 10 is the same as Fig. 8 but with linear coupling (see Eqs. (31) and (33)), and considering only the dipole-field regions of the SPS. It is seen that in this case the nominal intensity can be reached already for a chromaticity of 0.34 instead of 0.88 without linear coupling. Considering the same impedance in the horizontal and vertical planes of the regions without dipole fields and in the vertical plane of the dipole-field region, and zero in the horizontal plane of the dipole-field regions, yields Fig. 11. The beneficial effect of linear coupling is also observed, but slightly less than in Fig. 10, since 1/3 of the circumference has now a horizontal impedance. It is seen that in this case the nominal intensity can be reached already for a chromaticity of 0.52 instead of

0.88 without linear coupling, and 0.34 considering only the dipole-field regions.

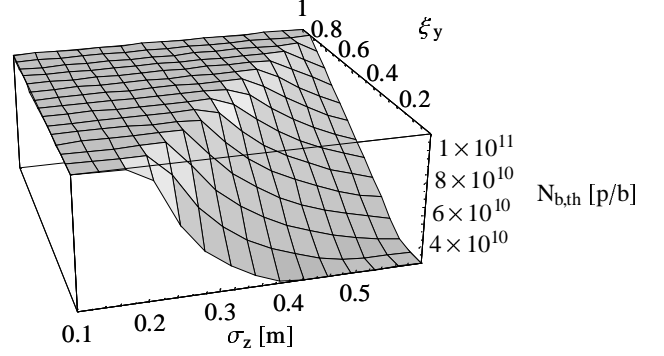


FIGURE 10. Same as Fig. 8, but in the presence of linear coupling and considering only dipole-field regions.

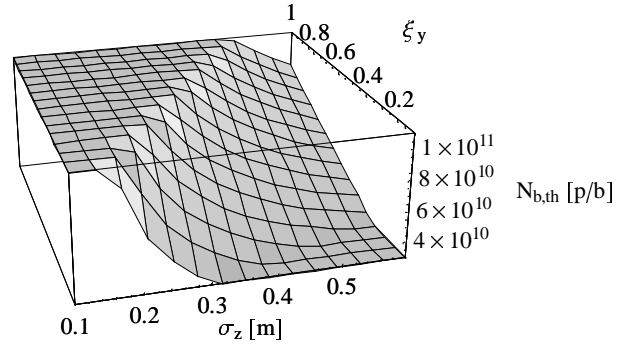


FIGURE 11. Same as Fig. 8, but in the presence of linear coupling and considering the dipole-field regions (2/3 of the SPS circumference) and the regions without dipole field (1/3 of the SPS circumference).

4 CONCLUSION

The equation used here for the classical transverse mode-coupling instability is the same as from (i) the coasting-beam approach using the peak values of bunch current and momentum spread, (ii) Ruth and Wang fast blow-up theory, (iii) Kernel et al. post-head-tail formalism, and (iv) Zotter theory for zero chromaticity.

This formalism has been applied to the SPS with the vertical broad-band electron-cloud induced impedance, which depends on bunch length and intensity. It is found that higher intensity thresholds can be reached by (i) increasing the chromaticity, and/or (ii) decreasing the bunch length, and/or (iii) using linear coupling.

The predicted SPS rise-time and stabilising effect of chromaticity, are in quantitative agreement with the

observations (made up to $\sim 6 \times 10^{10}$ p/b). It is predicted that the nominal beam should be stable for a sufficiently large chromaticity (~ 1). However, beam losses may appear due to other phenomena. It is proposed to use linear coupling in the SPS to reduce the value of the chromaticity needed to stabilise the nominal beam for LHC.

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