On the Transparency of the Electron Cloud to Synchrotron Radiation

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Abstract

We study the interaction of the synchrotron radiation, produced by a relativistic particle in a bending magnet, with the electron cloud present in the same magnet. The cloud is described as a collisionless magnetized plasma of very low, but finite temperature. Expressions are derived for the spectral intensity of synchrotron radiation far from the particle, which in absence of a cloud reduce to the Schott spectrum of radiation in vacuum.

For typical cloud parameters – a rarefied plasma, we fully neglect the refraction and only take into account the damping of the extraordinary and ordinary plasma waves at frequencies near the first electron cyclotron resonance (wave lengths $\sim$ mm) via interaction with resonance electrons. This effect would be the strongest in the hypothetic case of electron beam and electron cloud, but is found to be weaker in the realistic case of positively charged beam particle (proton, positron). In the latter case, by taking Maxwellian velocity distribution of the electrons (r.m.s. velocity $v_e = \beta_e c$) and fully neglecting the ordinary wave (factor β_e), we demonstrate that the dominant effect is coupling of π- mode of the spontaneous radiation with the extraordinary plasma wave.

1 INTRODUCTION

The goal of this paper is to study whether synchrotron radiation generated in a LHC bending magnet can significantly affect the electron cloud present within the same magnet. We consider the radiation of a relativistic particle (also called "test" particle) with rest mass M, charge $\pm Z|e|$ and energy γMc^2 ($\gamma >> 1$) moving along the central trajectory of a bending magnet (field B_0, radius $\rho = Mc^2\beta\gamma/(Z|e|B_0)$) in presence of a non-relativistic electron plasma (electron cloud) surrounding the central trajectory. Since the length of the magnet is much larger than the formation length ρ/γ, we will assume that the whole plasma volume is illuminated by the same radiation spectrum and that the size of the electron cloud, both transversally and longitudinally with respect to the direction of propagation of the radiation, is much larger than the radiated wavelength.

Following mainly [1], in Section 2 we compute the spectral density of radiation at frequency ω, generated by the test particle as it traverses finite volume of cold electron plasma of very low density. By neglecting two-particle interactions, the test particle radiates as if it is in a free space, but the radiation decays as it propagates through the cloud.

The energy losses of the test particle are defined as the work per second done by the breaking force acting on the particle due to electromagnetic field produced by the particle itself:

$$\frac{\delta \mathcal{E}}{\delta t} = Ze(\boldsymbol{v}^{(t)} \cdot \boldsymbol{E}) \qquad (1)$$

where $\boldsymbol{v}^{(t)}$ is the test particle velocity vector; $\boldsymbol{E}$ is the field produced by the particle at its own position $\boldsymbol{r}'$ and e is the electron charge. One can think of the field $\boldsymbol{E}$ in (1) as the plane monochromatic wave which, far from the source (current density $\boldsymbol{j}^{(t)} = eZ\boldsymbol{v}^{(t)}$), coincides with the spontaneous synchrotron radiation. By neglecting all effects taking place at the plasma boundary, this wave within the plasma splits into two waves – ordinary ($-$) and extraordinary (+) one. Propagation of the two plasma waves is described in the so called quasi-linear (geometrical optics) approximation. We realize that the geometrical optics description is not correct within several wavelengths from the source, but it can still be used approximately (as this was done in [1]).

It is also assumed that the plasma is stationary in time, i.e. it has no unstable (growing with time) modes even at the (low) frequencies near the electron cyclotron resonance.

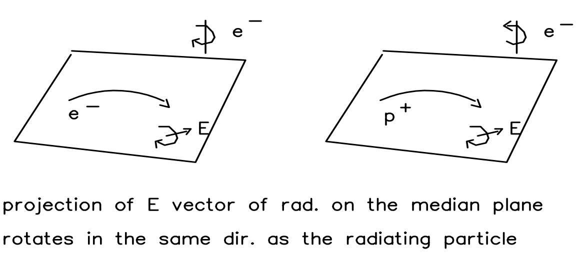
In the limit of zero plasma density, or negligible damping of the waves at frequency ω, i.e. $k''_{\pm}(\omega)L << 1$; $e^{-k''_{\pm}(\omega)L} \approx 1$; where L is the length traversed by the radiation within the plasma and $k''_{\pm}$ are absorption coefficients, our result should reduce to the usual formula of Schott for the spontaneous synchrotron radiation spectrum (as in vacuum).

By expanding the exponent: $e^{-k''_{\pm}(\omega)L} \approx 1 - k''_{\pm}(\omega)L$, the correction to the spontaneously radiated power is proportional to $-k''_{+}(\omega)L$, while the absolute value $k''_{+}(\omega)L$, multiplied by the spontaneously radiated power and integrated over ω and the angles, gives the total power deposited in the cloud. The latter quantity as considered in this work to be an adequate measure for the strength of interaction between synchrotron radiation and plasma.

<u>The two cases – negative (e^-) and positive (e^+, or p^+) radiating particle</u>

If the test particle is an electron in vacuum, a remote observer whose radius vector describes an angle θ with the external magnetic field ($\theta < \pi/2$ means above the median plane), sees elliptically polarized spontaneous synchrotron radiation wave with electric vector $\vec{E}$ rotating in the same direction as the electron ([5]). More precisely, the projection of $\vec{E}$ on the median plane rotates in the same direction as the electron. This remains true for both an observer located above or below the median plane – the polarization changes from left- to right-, or reverse when θ crosses $\pi/2$. Thus in this case $\vec{E}$ rotates synchronously with the plasma extraordinary wave, implying stronger interaction.

If the test particle is a positron or proton, then compared to the electron case, $\vec{E}$ reverses its orientation $\vec{E} \to -\vec{E}$, but still rotates in the same direction as the test particle, i.e. opposite to the electrons of the cloud and synchronously with the ordinary wave.



Parameters

L – propagation length of the radiation within the cloud;

B_0 – external magnetic field;

ω and $\lambda \equiv 2\pi c/\omega$ – frequency and wavelength of radiation far from the plasma;

N_0 – the number of electrons per cubic cm;

$\omega_p = \left(4\pi N e^2/m_e\right)^{1/2} = 5.64 \times 10^4 [s^{-1}] \sqrt{N_0[cm^{-3}]}$ – the electron plasma frequency;

$\Omega_e = \frac{eB_0}{m_e c} = 1.76 \times 10^7 [s^{-1}] B_0\ [Gs]$ – cyclotron frequency of the electrons;

$\Omega = \frac{ZeB_0}{Mc\gamma}$ – cyclotron frequency of the relativistic LHC particle ($\gamma >> 1$);

$n = \omega/\Omega_e$ – harmonic number;

$q = (\omega_p/\Omega_e)^2$ – density parameter;

$v_e \equiv \sqrt{< v^2 > /3}$ – the r.m.s. thermal velocity of the electrons in case of Gaussian distribution function:

$$f_e = N_0 \left(\frac{m_e}{2\pi T_e}\right)^{3/2} e^{-mv^2/2T_e}, \qquad (2)$$

where $T_e \equiv m_e v_e^2 = k_B T[K]$; $\beta_e \equiv v_e/c\, (1/\beta_e^2 = 5.11 \times 10^5/T_e\ [eV])$;

$r_D \equiv v_e/\omega_p$ – the Debye length;

y_0 – the distance of the electronic gyro-frequency to the critical frequency of the spontaneous synchrotron radiation spectrum:

$$y_0 \equiv \frac{2\Omega_e}{3\Omega\gamma^3} = \frac{2}{3\gamma^2}\ \frac{M}{Z\ m_e}. \qquad (3)$$

For electron rings ($|Z| = 1$, $\frac{M}{m_e} = 1$), y_0 is small: $\sim \gamma^{-2}$. For protons in the LHC ($M = m_p$; $\frac{m_e}{m_p} = 5.4 \times 10^{-4}$), $y_0 << 1$ both at injection ($\gamma = 480,\ y_0 = 1/188$) and collision ($\gamma = 7462,\ y_0 = 2 \times 10^{-5}$). For heavier radiating particles (like $^{82+}_{208}Pb$ ions), with mass $M = A \times m_p$, assuming the rigidity ρH is the same as for protons, the Lorenz factor γ is multiplied by Z/A, so the y_0 values for protons are multiplied by $(A/Z)^3$. One has: at injection ($\gamma = 189, y_0 \approx 0.1$) and collision ($\gamma = 2942,\ y_0 \approx 3 \times 10^{-4}$).

Estimation of the effect

The fraction of power deposited in the cloud relative to total power radiated can be estimated in the following way. We take: $N_0 = 10^6\ cm^{-3}$, $\lambda \sim 1\ mm$, $q = 10^{-8}$, $y_0 \sim 10^{-3}$ and thickness of the plasma slab $L = 10\ m$. For Maxwellian plasma, the order of magnitude of the absorption coefficients is known: $k_+^{''} L \approx \frac{q}{\beta_e}\frac{L}{\lambda}$, which should be multiplied by y_0 (the center of the absorption line) and by β_e (its width) to get:

$$\frac{q}{\beta_e}\frac{L}{\lambda}\ y_0\ \beta_e\ \sim 10^{-7}.$$

2 CORRECTIONS TO THE SPONTANEOUS SYNCHROTRON RADIATION SPECTRUM CAUSED BY WAVE ABSORPTION IN THE ELECTRON CLOUD

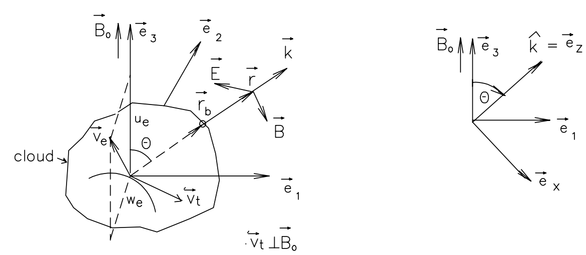


Figure 1: Left: test particle with velocity vector $\vec{v}_t \perp \vec{B}_0$ traversing electron plasma and velocity vector $\vec{v}_e$ of an electron of the plasma. Right: coordinate frame $\vec{e_x}, \vec{e_y}, \vec{e_z}$. obtained by rotation of $\vec{e_1}, \vec{e_2}, \vec{e_3}$ around the $\vec{e_2}$ axis at angle $-\theta$, so that the direction of propagation of radiation is along $\vec{e_z}$ ($\vec{k} \| \vec{e_z}$)

2.1 *The self-consistent equations describing small plasma oscillations and the dispersion relation ([1], [2], [7])*

We mainly follow [1], where the energy losses due to synchrotron radiation are studied for a slab of quasi-relativistic uniform electron plasma (the thermo-nuclear reactor). The electric field within the plasma $\boldsymbol{E}$, which corresponds to an external current, or a test particle current $j^{(t)}$, satisfies the

Maxwell equation:

$$rot\ rot\ \boldsymbol{E} + \frac{1}{c^2}\frac{\delta^2 \boldsymbol{E}}{\delta t^2} + \boldsymbol{j}^{(abs)} = \boldsymbol{j}^{(\mathrm{t})}, \qquad (4)$$

where the current density $\boldsymbol{j}^{(abs)}$, caused by the field $\boldsymbol{E}$, describes absorption and induced radiation within the plasma.

A test particle traversing the plasma is shown on Fig. 1. We have chosen the direction of the external field $\vec{B}_0$ to be parallel to the $\vec{e}_3$ axis and the radius vector $\vec{r}$ of the remote observer to lie in the plane $\vec{e}_1, \vec{e}_3$.

In what follows we use the relativistic form of the dielectric permittivity tensor (as in [1]; see also [8], [9]) to compute the Fourier components of the relativistic test-particle current. The same tensor, but taken in a non-relativistic approximation, ([2], [4]) will later be used to describe small oscillations of the electron plasma.

For a particle with mass of rest M and charge $q = Ze$, the relativistic tensor is:

$$\boldsymbol{Q} = -\frac{4\pi q^2}{m}\int d^3v\, f(\vec{v})\,\frac{1}{\gamma}\times \times\left(\frac{\omega - k_\parallel u/\gamma}{w}\,\frac{\partial f}{\partial w} - \frac{k_\parallel}{\gamma}\frac{\partial f}{\partial u}\right) \sum_{n=-\infty}^{n=+\infty}\frac{\boldsymbol{T}_n(\vec{v})}{\omega - (k_\parallel u/\gamma) - n\Omega} -\frac{4\pi q^2}{m}\int d^3v\, f(\vec{v})\frac{u}{w\gamma}\left(w\frac{\partial f}{\partial u} - u\frac{\partial f}{\partial w}\right)\boldsymbol{e_3 e_3} \qquad (5)$$

$$\boldsymbol{T}_n(\vec{v}) = \begin{pmatrix} \frac{n^2w^2}{z^2}J_n^2 & \frac{inw^2}{z}J_nJ_n' & \frac{uwn}{z}J_n^2 \\ \frac{-inw^2}{z}J_nJ_n' & w^2{J'}_n^2 & -iuwJ_nJ_n' \\ \frac{uwn}{z}J_n^2 & iuwJ_nJ_n' & u^2J_n^2 \end{pmatrix}$$

with $J_n(z)$ being the Bessel function; $z \equiv k_\perp w/(\Omega\gamma)$; $\vec{k} = k_\perp\vec{e}_1 + k_\parallel\vec{e}_3$ (see Fig 1) and w and u denoting the components of the vector $\vec{v}$ transverse and parallel to $\vec{B}_0$.

Above $f(\vec{v})$ is relativistic distribution function, normalized so that $\int d^3v f(\vec{v}) = 1$. The argument $\vec{v}$ (notice an unusual notation!) denotes the particle momentum divided by the mass of rest, i.e. $\vec{v} \equiv \beta c\gamma$, with γ being the usual relativistic factor: $\gamma \equiv (1-\beta^2)^{-1/2} = (1+v^2/c^2)^{1/2}$. In the non-relativistic case ($\gamma \to 1$), $\vec{v}$ becomes the particle velocity.

By taking plane monochromatic waves $\boldsymbol{E} = \boldsymbol{E}_{\vec{k},\omega}\ e^{i(\vec{k}\cdot\vec{r})-i\omega t}$, $\boldsymbol{j}^{(t)} = \boldsymbol{j}^{(t)}_{\vec{k},\omega}\ e^{i(\vec{k}\cdot\vec{r})-i\omega t}$ (the size of the plasma volume is much larger than the wavelengths of interest), the Fourier-transform of (4) reads:

$$\boldsymbol{R}^{(e)}\cdot\boldsymbol{E}_{\vec{k},\omega} = \boldsymbol{j}^{(t)}_{\vec{k},\omega},$$
$$\boldsymbol{R}^{(e)} \equiv (c^2k^2 - \omega^2)\boldsymbol{I} - c^2\boldsymbol{kk} + \boldsymbol{Q}^{(e)}. \qquad (6)$$

where $\boldsymbol{I}$ is the unity 3×3 matrix, $\boldsymbol{R}^{(e)}$ and $\boldsymbol{Q}^{(e)}$ are functions of real $\boldsymbol{k}$ and ω and $\boldsymbol{Q}^{(e)}$ is obtained from $\boldsymbol{Q}$ in (5) with substituting the electron parameters e, m_e, $\vec{v}_e$, $f(\vec{v}_e)$, Ω_e.

In the rotated coordinate frame $\vec{e_x}, \vec{e_y}, \vec{e_z}$:

$$(c^2k^2-\omega^2)\boldsymbol{I} - c^2\boldsymbol{kk} = -\omega^2\begin{pmatrix}1-\mathcal{N}^2 & 0 & 0\\ 0 & 1-\mathcal{N}^2 & 0\\ 0 & 0 & 1\end{pmatrix};$$

$$\mathcal{N} \equiv kc/\omega\ .$$

For a Maxwellian plasma, the non-relativistic approximation of $\boldsymbol{Q}^{(e)}$ near the first cyclotron resonance is discussed in Section 3.

The dispersion equation of small plasma oscillations is:

$$\Delta(k) \equiv \det\boldsymbol{R}^{(e)} = 0. \qquad (7)$$

For a fixed real ω, it can be shown ([1]) that in the limit of rarefied plasma $\omega_p << \omega$, the dispersion equation (7) becomes biquadratic with respect to k, so there are only two solutions for k^2, denoted here by $k^2_\pm$, corresponding to the ordinary and extraordinary waves (refraction index values $\mathcal{N}_\pm \equiv k_\pm c/\omega$). Thus we have in this limit:

$$\Delta(k) = c^4(k^2 - k_+^2)(k^2 - k_-^2). \qquad (8)$$

2.2 *The field propagator*

Consider a statistical ensemble (the beam) of test particles with coordinates $\vec{r'}$, velocities $\vec{v'}$. One can introduce macroscopic fluctuating-current densities $j(t, \vec{r'})$, functions of time and $\vec{r'}$. To find the field radiated at frequency ω by a current fluctuation $j(t,\vec{r'})$, one has to invert (6) and then carry out contour integration over k and spatial integration over all sources $\vec{r'}$. The result is [1]: [1]

$$\begin{aligned}\boldsymbol{E}_\omega(\boldsymbol{r}) &= \int d^3r'\ \boldsymbol{W}\cdot\boldsymbol{j}^{(t)}_\omega(\vec{r'}),\ \text{with}\\ \boldsymbol{W} &= \frac{i\omega e^{i(\omega/c)r - i(\omega/c)\hat{r}\cdot\vec{r'}}}{c^4 r(k_+^2 - k_-^2)} \left(\boldsymbol{\lambda}_+^{(e)}e^{-k''_+ r_b} - \boldsymbol{\lambda}_-^{(e)}e^{-k''_- r_b}\right),\\ \hat{r} &\equiv \vec{r}/r, \end{aligned} \qquad (9)$$

where the elements of the matrix $\boldsymbol{\lambda}^{(e)}$ are the cofactors of $\boldsymbol{R}^{(e)}$, i.e. $(R^{(e)})^{-1} = \boldsymbol{\lambda}^{(e)}/\Delta$ and r_b denotes a point at the plasma boundary.

The indices $+$ and $-$ appear because during the contour integration the argument k in $\lambda^{(e)}_{i,j}(k,\omega)$ is substituted with $k_\pm$, where $k_\pm$ are the two roots out of four having positive imaginary part. The factor $(k_+^2 - k_-^2)$ in the denominator appears since, according to (8), $d\Delta/dk|_{k=k_\pm} = \pm 2c^4k_\pm(k_+^2 - k_-^2) \sim \pm 2\omega c^3(k_+^2 - k_-^2)$.

In the limit $\omega_p << \omega$ (same as for (8)), it can be shown that:

$$\boldsymbol{\lambda}_+^{(e)} - \boldsymbol{\lambda}_-^{(e)} = c^2(k_+^2 - k_-^2)(\boldsymbol{I} - \hat{r}\hat{r}), \qquad (10)$$

[1] This expression for the field has the correct asymptotic at $r\to\infty$. It has been assumed valid also in the vicinity of the source $\vec{r'}$.

where in the rotated frame $\vec{e_x}, \vec{e_y}, \vec{e_z}$, shown on Figure 1:

$$(\boldsymbol{I} - \hat{\boldsymbol{r}}\hat{\boldsymbol{r}}) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

If we further take $e^{-k''_{\pm} r_b} \to 1$, then W reduces to the vacuum propagator.

2.3 *The spectral density of radiation* S_ω

The power radiated in direction $\hat{r} = \vec{r}/r$ per unit area and per unit frequency interval is [1] (the bar indicates statistical average):

$$\begin{aligned} S_\omega &= \frac{c}{8\pi^2} \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} \overline{\boldsymbol{E}^*(\vec{r}, t) \cdot \boldsymbol{E}(\vec{r}, t+\tau)} \\ &= \frac{c}{8\pi^2} \overline{\boldsymbol{E}_{-\omega}(\vec{r}) \cdot \boldsymbol{E}_\omega(\vec{r})}. \end{aligned} \tag{11}$$

We substitute here (9) and notice that the only dependence on $\vec{r'}$ is in the factor $e^{i(\omega/c)\hat{r}\cdot\vec{r'}}$. S_ω therefore contains the expression:

$$\int d^3r' \int d^3r'' \int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} e^{i(\omega/c)\hat{r}\cdot(\vec{r'} - \vec{r''})} \times \overline{\boldsymbol{j}^{(t),*}(\vec{r'}, t) \cdot \boldsymbol{j}^{(t)}(\vec{r''}, t+\tau)}\ , \tag{12}$$

where the advance from $\vec{r'}$ to $\vec{r''}$, during time interval τ, is along the unperturbed trajectory of the test particle. The above expression (12) (as a function of real ω) is equal to the spectral density $(\boldsymbol{j}^{(t)^*}\boldsymbol{j}^{(t)})_{\vec{k}=\frac{\omega}{c}\hat{r},\omega}$ of current fluctuations for a plasma in an external magnetic field, [1], [2]. It is also called non-interacting current correlator. Thus we obtain for S_ω:

$$S_\omega = \frac{c}{8\pi^2} Sp\, \boldsymbol{W}^+ \cdot (\boldsymbol{j}^{(t)\star}\boldsymbol{j}^{(t)})_{\vec{k}=\frac{\omega}{c}\hat{r},\omega} \cdot \boldsymbol{W} \tag{13}$$

2.4 *The spectral density of test particle current*

The spectral density $\boldsymbol{j}^{(t)}(\vec{k} = \frac{\omega}{c}\hat{r}, \omega)$, or correlator $(\boldsymbol{j}^{(t)\star}\boldsymbol{j}^{(t)})_{\vec{k}=\frac{\omega}{c}\hat{r},\omega}$, can be obtained either directly, by Fourier-expanding the unperturbed test-particle motion, or by applying the dissipation-fluctuation theorem, [1],[2]. The latter theorem states that it equals the anti-hermitian part of the tensor $\boldsymbol{Q}$ defined above, but written for an ensemble of test particles instead of electrons. Also, according to the same theorem, $\boldsymbol{Q}^{(t)}$ should be taken in the limit of vanishing particle-particle interactions, so one has to use a vanishing imaginary part $\omega \to \omega - i\varepsilon$. Correspondingly, we replace the parameters q, m, $\vec{v}$, $f(\vec{v})$, Ω in (5) with the ones describing an "ensemble" of a single test particle with charge Ze, mass Am, momentum $m\vec{v}_t$, relativistic distribution function $f(\vec{v_t'}) = \delta(\vec{v_t'} - \vec{v}_t)$ and cyclotron frequency $\Omega = ZeB_0/(Amc\gamma_t)$. The result is:

$$\begin{aligned} &(\boldsymbol{j}^{(t)\star}\boldsymbol{j}^{(t)})_{\vec{k}=\frac{\omega}{c}\hat{r},\omega} = 1/2[\boldsymbol{Q}^{(t)} - \boldsymbol{Q}^{(t)+}]|_{\vec{k}=\frac{\omega}{c}\hat{r},\omega} = \\ &= \frac{2\pi(Ze)^2}{\gamma_t^2} \sum_{n=-\infty}^{+\infty} \delta(\omega - n\Omega)\, \boldsymbol{T}_n(\vec{v}_t)|_{\vec{k}=\frac{\omega}{c}\hat{r},\omega=n\Omega}, \end{aligned} \tag{14}$$

where use has been made of the formal equality:

$$\lim_{\varepsilon\to 0} Im \frac{1}{\omega - i\varepsilon - n\Omega} = -\pi\delta(\omega - n\Omega). \tag{15}$$

To simplify the tensor $\boldsymbol{T}_n(\vec{v_t})$, we first notice that $u_t = 0$, hence the elements in the third row and column of $\boldsymbol{T}_n(\vec{v_t})$ are zero. The remaining 2x2 part of the tensor, transformed in the rotated coordinate frame $\vec{e_x}, \vec{e_y}, \vec{e_z}$, is

$$\begin{aligned} &\boldsymbol{T}_n(\vec{v}_t)|_{\vec{k}=\frac{\omega}{c}\hat{r},\omega} \\ &= \begin{pmatrix} \frac{n^2 w_t^2}{z^2} J_n^2 \cos^2\theta & \frac{inw_t^2}{z} J_n J_n' \cos\theta \\ \frac{-inw_t^2}{z} J_n J_n' \cos\theta & w_t^2 {J'_n}^2 \end{pmatrix}_{\vec{k}=\frac{\omega}{c}\hat{r},\omega} = \\ &= \begin{pmatrix} j_{x,n}^2 & ij_{x,n}j_{y,n} \\ -ij_{x,n}j_{y,n} & j_{y,n}^2 \end{pmatrix} = \vec{j_n}^{\,*}\vec{j_n}, \end{aligned}$$

where

$$\vec{j_n} \equiv \begin{pmatrix} j_{x,n} \\ ij_{y,n} \end{pmatrix}; \tag{16}$$

$$\begin{aligned} j_{x,n} &\equiv j_{\pi,n} = \frac{nw_t}{z} J_n \cos\theta = \frac{n\Omega\gamma_t}{k_\perp} J_n \cos\theta \\ &= \frac{n\Omega\gamma_t}{\omega/c} \text{ctg}\ \theta J_n\ = c\gamma_t\ \text{ctg}\ \theta\ J_n; \\ j_{y,n} &\equiv j_{\sigma,n} = w_t J'_n = c\beta_t\gamma_t J'_n \end{aligned} \tag{17}$$

and the argument is

$$z = \omega w_t \sin\theta/(\gamma_t c\Omega) = n w_t c \sin\theta/\gamma_t = n\beta_t \sin\theta$$

(we have replaced ω with $n\Omega$ and used that $k_\parallel = \omega\cos\theta/c$ and $k_\perp = \omega\sin\theta/c$).

We will see (the Schott formula below) that $j_{\pi,n}$ and $j_{\sigma,n}$ are actually proportional to the π and σ-component of linear polarization of the electric vector of spontaneous radiation (harmonic n). Thus the electric vector is parallel to $\vec{j_n}$ and elliptically polarized ($|j_{\pi,n}| \neq |j_{\sigma,n}|$), with direction of rotation, left- or right- as given by the signs of the components of $\vec{j_n}$. For any n, these two components have equal sign for $\theta > 0$, and opposite signs if $\theta \to \pi - \theta$, i.e. the direction of polarization of the n-th harmonic is reversed. If one fixes the frequency seen by the observer to a real positive value $\omega > 0$, then for an observer above the median plane $\Omega > 0$, which means that positive n have to be taken in the sum because of the δ-function. Below the median plane ($\theta \to \pi - \theta$), Ω is negative and hence negative n values have to be taken, which leads both $j_{\pi,n}$ and $j_{\sigma,n}$ reversing their signs.

For S_ω we get from (13), (9) and (14) :

$$\begin{aligned} r^2 S_\omega &= \frac{\omega^2(Ze)^2}{4\pi c^7\gamma_t^2} \sum_{n=-\infty}^{+\infty} \delta(\omega - n\Omega) \frac{Sp}{|k_+^2 - k_-^2|^2} \\ &\quad \left(\boldsymbol{\lambda}_+^{(e)} e^{-k''_+ r_b} - \boldsymbol{\lambda}_-^{(e)} e^{-k''_- r_b}\right)^+ \cdot \boldsymbol{T}_n(\vec{v_t}) \cdot \\ &\quad \cdot \left(\boldsymbol{\lambda}_+^{(e)} e^{-k''_+ r_b} - \boldsymbol{\lambda}_-^{(e)} e^{-k''_- r_b}\right) \end{aligned} \tag{18}$$

2.5 The Schott formula

Here we derive the spectral density S^0_ω of spontaneous radiation of the test particle (as in vacuum, no cloud), emitted at angle θ with respect to the external magnetic field, called the Schott formula [5].

If the size of the plasma is much smaller than the absorption depth ($k''_\pm r_b << 1$) then, by taking into account (10), (18) becomes:

$$\begin{aligned} r^2 S^0_\omega &= \frac{\omega^2 (Ze)^2}{4\pi c^3 \gamma_t^2} \sum_{n=-\infty}^{+\infty} \delta(\omega - n\Omega)(j^2_{\pi,n} + j^2_{\sigma,n}) \\ &= \frac{\omega^2 (Ze)^2}{4\pi c^3 \gamma_t^2} \sum_{n=-\infty}^{+\infty} \delta(\omega - n\Omega)(c^2\gamma_t^2 \operatorname{ctg}^2\theta\, J_n^2(z) \\ & \quad + w_t^2 {J'_n}^2(z)) = \\ &= \frac{(Ze)^2\Omega^2}{2\pi c} \sum_{n=1}^{\infty} n^2 \left(\operatorname{ctg}^2\theta J_n^2(z) + \beta_t^2 {J'}_n^2(z)\right) \\ & \quad \delta(\omega - n\Omega), \end{aligned} \qquad (19)$$

where $z = n\beta_t \sin\theta$. The term with $n = 0$ does not contribute and the terms with n and $-n$ are equal, giving a factor 2. In (19), $r^2 S^0_\omega$ is the the energy/sec, radiated at angle θ with respect to the external magnetic field, per unit solid angle and per unit frequency interval.

2.6 Integration of the Schott spectrum over frequencies and angles

We follow the standard integration procedure ([5], [6]) to obtain the total power radiated by the test particle (from now on we omit the subscript "t"). For a highly relativistic such particle $\gamma >> 1$, $\beta \approx 1$, the radiation is concentrated near the median plane: $\theta \approx \pi/2$. The order n of the Bessel functions is therefore nearly equal to their argument: $z = n\beta \sin\theta \sim n$ and one can use the asymptotic formulas:

$$\begin{aligned} J_n(z) &= \frac{\epsilon^{1/2}}{\pi\sqrt{3}} K_{1/3}\left(\frac{n}{3}\,\varepsilon^{3/2}\right), \\ J'_n(z) &= \frac{\varepsilon}{\pi\sqrt{3}} K_{2/3}\left(\frac{n}{3}\,\varepsilon^{3/2}\right), \end{aligned} \qquad (20)$$

where $\varepsilon = 1 - z^2/n^2 = 1 - \beta^2 \sin^2\theta \; << 1$. We will only need the above expressions for large harmonics $n >> 1$, where the sum over n can be replaced by an integral, which is done by the following transformation from $n,\ \theta$ to new variables y, ψ:

$$\begin{aligned} &\psi = \gamma\cos\theta; && y = \frac{2}{3}\, n\gamma^{-3}; \\ &d\psi = \gamma\, d\cos\theta; && dy = \frac{2}{3}\, dn\gamma^{-3} \quad (dn = 1); \\ &-\gamma \;<\; \psi \;<\; \gamma; && 0 < y < \infty. \end{aligned}$$

Here y measures the relative distance to the critical harmonic $\frac{3}{2}\gamma^3$ (nearly equal to the spectrum maximum), while ψ measures the angle between the direction vector of propagation of the radiation and the horizontal plane (in units of $1/\gamma$). In the arguments of K, ε is expanded over the small quantities $\cos\theta$ and $\sqrt{\varepsilon}_0 \equiv 1/\gamma$ and by keeping only terms of the order of $1/\gamma^2$:

$$\begin{aligned} &\varepsilon = \varepsilon_0 \left[1 + \left(\frac{\cos\theta}{\sqrt{\varepsilon}_0}\right)^2 + \ldots \right] \approx \varepsilon_0 \left(1 + \psi^2\right) = \\ &\quad = \frac{1}{\gamma^2}\left(1 + \psi^2\right) \\ &\operatorname{ctg}\theta \approx \cos\theta = \frac{\psi}{\gamma}. \end{aligned}$$

By substituting (20) into (19) and integrating over angles and frequencies, the total power radiated W_0 is (here $\rho = c/\Omega$):

$$\begin{aligned} &2\pi \int_0^\infty d\omega \int_0^\pi d\theta \sin\theta\, r^2 S^0_\omega(\theta) = \\ &= \frac{(Ze)^2 c}{3\pi^2\rho^2} \int_0^\pi d\theta \sin\theta \sum_{n=1}^{\infty} n^2 \\ &\left[ctg^2\theta\varepsilon K^2_{1/3}\left(\frac{n}{3}\varepsilon^{3/2}\right) + \varepsilon^2 K^2_{2/3}\left(\frac{n}{3}\varepsilon^{3/2}\right)\right] = \\ &= \frac{27}{16\,\pi^2} W_0 \int_0^\infty y^2 dy \int_{-\gamma}^{+\gamma} d\psi \\ &\left[\psi^2(1+\psi^2) K^2_{1/3}(\eta) + (1+\psi^2)^2 K^2_{2/3}(\eta)\right] = \\ &= W_0 \left(\frac{1}{8} + \frac{7}{8}\right) = W_0, \end{aligned} \qquad (21)$$

where $\eta = \frac{1}{2}\, y\, (1 + \psi^2)^{3/2}$ and $W_0 = \frac{2}{3}\frac{Z^2 e^2 c}{\rho^2}\gamma^4$. ($\gamma$ is replaced with infinity in the upper limit of integration over ψ, because the K functions are nonzero only for argument of the order of unity).

The frequency radiated ω $(= n\Omega)$, which corresponds maximum of the spontaneous synchrotron radiation spectrum is $\simeq (3/2)\Omega\gamma^3$ meaning that the expression under the integral sign in (21), as a function of $y \equiv \frac{2}{3}\frac{\omega}{\Omega\gamma^3}$, reaches its maximum at $y \simeq 1$.

The π mode (first term) is radiated in directions above and below the median plane and becomes zero in the plane (the factor ψ^2). For the σ mode, the radiation is centered in the median plane and its total contribution is 7 times larger.

3 ESTIMATION OF THE ABSORBED POWER FOR A MAXWELLIAN CLOUD

3.1 Wave absorption at frequencies near the first cyclotron resonance

We consider a rarefied plasma $q << 1$ with electronic temperature $T_e \sim 100\ eV$ ($\beta_e \equiv \bar{v}_e/c \sim 0.01$). We assume that:

$$\left(\frac{\omega_p}{\omega}\right)^2 \frac{1}{\beta_e} = \frac{q}{n^2\beta_e} << 1 \qquad (22)$$

is fulfilled for all harmonics n of $\omega = n\ \Omega_e$, even at the cyclotron resonance $n = 1$. For frequencies in the vicinity of the first cyclotron resonance, the dielectric tensor (5) has the form ([2], [7]):

$$Q = \begin{pmatrix} -\frac{q}{4}+\sigma & \frac{iq}{4}+i\sigma & 0 \\ \frac{-iq}{4}-i\sigma & -\frac{q}{4}+\sigma & 0 \\ 0 & 0 & -q \end{pmatrix} \qquad (\sigma >> q)$$

$$\sigma = i\sqrt{\frac{\pi}{8}}\frac{\omega_p^2}{\omega^2\beta_e\cos\theta}\, w(z_1),$$
$$w(z_1) = e^{-z_1^2}\left(\frac{\cos\theta}{|\cos\theta|}+\frac{2i}{\sqrt{\pi}}\int_0^{z_1} e^{y^2}dy\right), \tag{23}$$

where $w(z)$ is the probability integral (error function of complex argument).

In more details, for Maxwellian distribution, the n-th member of the sum in (5) ($n = \pm 1 \pm 2 \ldots$) is proportional to $e^{-z_n^2}$ with $z_n \equiv \frac{\omega - n|\Omega_e|}{\sqrt{2}\omega\beta_e\cos\theta}$. As ω approaches $|\Omega_e|$, the contribution to the tensor of the member (term) with $n = 1$ is the largest since $e^{-z_1^2} \sim 1$. This term is is caused by "normal Doppler effect", i.e. presence of electrons rotating in the same direction as the ω-harmonic and with velocities nearly equal to its phase velocity (for which $\omega - |\Omega_e| \approx (u/c)\,\omega\cos\theta$). For the other members of the sum, including the one with $n = -1$, produced by a harmonic rotating opposite to the electrons, $|z_n| >> 1$, so their contribution is exponentially small (their total contribution is $\sim q$). The picture is the same for higher resonances with $|\sigma_n|$ rapidly decreasing (roughly as $1/n!$).

By keeping only the resonance σ terms, the tensor Q becomes 2-dimensional and after transforming it into to frame $\vec{e_x}, \vec{e_y}, \vec{e_z}$ and substituting it into (6), one gets:

$$\boldsymbol{R}^{(e)} = \omega^2 \begin{pmatrix} \mathcal{N}^2 - 1 + \sigma\cos^2\theta & i\sigma\cos\theta \\ -i\sigma\cos\theta & \mathcal{N}^2 - 1 + \sigma \end{pmatrix}, \tag{24}$$

where

$$\mathcal{N} \equiv kc/\omega\,.$$

The determinant is ($\mathcal{N}_\pm \equiv k_\pm c/\omega$):

$$\Delta = \omega^4(\mathcal{N}^2 - \mathcal{N}_+^2)(\mathcal{N}^2 - \mathcal{N}_-^2), \tag{25}$$

where the roots are:

$$\mathcal{N}_-^2 = 1; \quad \mathcal{N}_+^2 = 1 - \sigma(1+\cos^2\theta). \tag{26}$$

As expected, in our approximation the ordinary wave propagates as in vacuum. The matrices $\lambda_\pm^{(e)}$ are computed by taking $\lambda^{(e)} \equiv (R^{(e)})^{-1}\,\Delta$ and substituting there $\mathcal{N}_\pm$ from (26). The result is:

$$\boldsymbol{\lambda}_+^{(e)} = \omega^2\sigma \begin{pmatrix} -\cos^2\theta & -i\cos\theta \\ i\cos\theta & -1 \end{pmatrix}, \tag{27}$$

$$\boldsymbol{\lambda}_-^{(e)} = \omega^2\sigma \begin{pmatrix} 1 & -i\cos\theta \\ i\cos\theta & \cos^2\theta \end{pmatrix}, \tag{28}$$

which can also be written as:

$$\boldsymbol{\lambda}_\pm^{(e)} = \vec{e}_\pm \vec{e}_\pm^{\,*}\, Sp\,\lambda_\pm^{(e)}; \qquad Sp\,\lambda_\pm^{(e)} = \mp(1+\cos^2\theta), \tag{29}$$

where the eigen-vectors are:

$$\vec{e}_+ \equiv \frac{\begin{pmatrix} -\cos\theta \\ i \end{pmatrix}}{\sqrt{1+\cos^2\theta}} \quad \vec{e}_- \equiv \frac{\begin{pmatrix} 1 \\ i\cos\theta \end{pmatrix}}{\sqrt{1+\cos^2\theta}}. \tag{30}$$

Thus the columns of $\boldsymbol{\lambda}_\pm^{(e)}$ are proportional to the components of the electric field vectors $\vec{E}_\pm$ of the two eigensolutions called extraordinary (+) and ordinary (−) plasma waves. The extraordinary wave electric field vector $\vec{E}_+$ is parallel to $\vec{e}_+$ and rotates in the same direction as the electrons.

It can be shown [2], that (29) is always fulfilled for nearly transparent media (when the anti-hermitian part of $\boldsymbol{Q}$ is small compared to its hermitian part) .

One can also check directly that (10) is indeed fulfilled:

$$\frac{1}{\omega^2}\,\frac{\boldsymbol{\lambda}_+^{(e)} - \boldsymbol{\lambda}_-^{(e)}}{\mathcal{N}_+^2 - \mathcal{N}_-^2} = \frac{\boldsymbol{\lambda}_+^{(e)} - \boldsymbol{\lambda}_-^{(e)}}{c^2(k_+^2 - k_-^2)} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \tag{31}$$

3.2 Estimation of the absorbed power

The roots $k_\pm$ and the cofactors $\boldsymbol{\lambda}_\pm$ are found in the previous section. By using some properties of $\boldsymbol{\lambda}_\pm$:

$$\boldsymbol{\lambda}_\pm.\boldsymbol{\lambda}_\mp = 0$$
$$\frac{\boldsymbol{\lambda}_+^+.\boldsymbol{\lambda}_+}{\omega^4\sigma^2} = (1+\cos^2\theta)\begin{pmatrix} \cos^2\theta & i\cos\theta \\ -i\cos\theta & 1 \end{pmatrix},$$
$$\frac{\boldsymbol{\lambda}_-^+.\boldsymbol{\lambda}_-}{\omega^4\sigma^2} = (1+\cos^2\theta)\begin{pmatrix} 1 & -i\cos\theta \\ i\cos\theta & \cos^2\theta \end{pmatrix},$$

and also the spectral density (16) of the test particle current $\vec{j_n}$, (18) is transformed as follows:

$$\begin{aligned} r^2 S_\omega =& \frac{\omega^2(Ze)^2}{4\pi c^7\gamma^2}\sum_{n=-\infty}^{+\infty}\frac{\delta(\omega - n\Omega)}{|k_+^2 - k_-^2|^2}\times \\ &\times Sp\left(\boldsymbol{\lambda}_+^+.\boldsymbol{\lambda}_+.\,e^{-2k_+^{''}r_b} + \boldsymbol{\lambda}_-^+.\boldsymbol{\lambda}_-.\,e^{-2k_-^{''}r_b}\right).\,\vec{j_n}^{\,*}\,\vec{j_n} \\ =& \frac{\omega^2(Ze)^2}{4\pi c^3\gamma^2}\sum_{n=-\infty}^{+\infty}\frac{\delta(\omega - n\Omega)}{1+\cos^2\theta}\times \\ &\times\Big[(j_{\pi,n}\cos\theta + j_{\sigma,n})^2\, e^{-2k_+^{''}r_b} \\ &+(j_{\pi,n} - j_{\sigma,n}\cos\theta)^2\, e^{-2k_-^{''}r_b}\Big] \end{aligned} \tag{32}$$

We will interpret (32), integrated over real and positive frequencies ω and angles $0 < \theta < \pi$, as the energy per second radiated by an *electron* in presence of *electron* plasma. If the beam particle and particles of the cloud has opposite sign, then the roles of ordinary extraordinary waves are reversed and correspondingly one has to exchange the indices $+$ and $-$ of $k^{''}$ in the two exponents.

With this in mind, we substitute r_b with the propagation length L within the cloud and expand the exponents $e^{-2k''_{\pm}L} \approx 1-2k''_{\pm}L$. This can only be done for small optical depth $k''_{+}L \sim \frac{2\pi qL}{\beta_e\lambda} << 1$, which is true for $\beta_e \sim 0.01$ and the parameters in Table 1. The unity produces the spontaneous spectrum (19) while the terms $-2k''_{\pm}L$ with their signs inverted yield:

$$\frac{\omega^2(Ze)^2}{2\pi c^3\gamma^2}\sum_{n=-\infty}^{+\infty}\frac{\delta(\omega-n\Omega)}{1+\cos^2\theta}\Big[(j_{\pi,n}\cos\theta+j_{\sigma,n})^2\,k''_{\pm}L + (j_{\pi,n}-j_{\sigma,n}\cos\theta)^2\,k''_{\mp}L\Big] \quad (33)$$

where the upper sign applies if a negatively charged beam travels through the electron cloud, and the lower sign refers to a positive beam charge as in the LHC, and

$$j_{\pi,n} = c\gamma\; ctg\;\theta\; J_n;\; j_{\sigma,n} = c\gamma J'_n,$$

$$\begin{aligned} k''_{+}(\omega,\theta)L =& \frac{\omega L}{2\,c}\,\mathrm{Im}\,\mathcal{N}_{+}^2 = \\ =& \frac{1}{4}\sqrt{\frac{\pi}{2}}\,\frac{\Omega_e^2}{\omega}\frac{qL}{\beta_e c}\,\frac{1+\cos^2\theta}{\cos\theta}\times \\ & \times\; \boldsymbol{e}^{-\left(\frac{\omega-|\Omega_e|}{\sqrt{2}\omega\beta_e\cos\theta}\right)^2} \sim \frac{qL}{\beta_e\lambda}; \\ k''_{-}(\omega,\theta)L \sim& \frac{qL}{\lambda}\,\boldsymbol{e}^{-\left(\frac{\omega-|\Omega_e|}{\sqrt{2}\omega\beta_e\cos\theta}\right)^2} \\ (\text{ where } \lambda \equiv& 2\pi c/\omega;\ \omega = n\Omega \approx \Omega_e). \end{aligned} \quad (34)$$

Since k''_{-} is β_e times smaller than k''_{+} we only show its order of magnitude. It is easy to compute it, if the q terms in $\boldsymbol{Q}$ are kept, [2].

We choose the cyclotron frequency of the test particle to be positive for an observer above the median plane $\Omega > 0$. Since $\omega > 0$, only positive n contribute. For each n, there are two contributions – scalar products (squared) between $\vec{j}_n$ and the unit vectors $\vec{e}_+$ and $\vec{e}_-$ of the counterclockwise and clockwise rotating plasma waves. Thus the elliptically polarized synchrotron radiation wave interacts with both extraordinary and ordinary plasma modes. This is because the elliptical polarization can be decomposed into a left- and right- circular polarizations.

1) In the hypothetic case – the test particle being an electron in electron cloud, the current $\vec{j}_\omega$ has counterclockwise polarization [2] and an observer located above the median plane ($\cos\theta > 0$), sees counterclockwise rotating both beam and plasma electrons.

By setting $k''_{-} = 0$ (taking only the first term), and noticing that n, $\cos\theta$, $j_{\pi,n}$ and $j_{\sigma,n}$ are all positive, we see that the contributions from σ and π modes add up (stronger absorption).

2) For a proton or positron in an electron cloud, by setting $k''_{-} = 0$ (taking only the second term), the contributions from the σ and π modes partially compensate each other. The factor $1/\cos\theta$ in k''_{+} is canceled and both modes participate with a factor $\cos\theta \sim \gamma^{-1}$. The absorption occurs away from the median plane (zero in the plane). For this case the integration is carried out below.

We take only the second term in (33) and integrate over angles and frequencies, same as this was done for (21) (here $W_0 y_0 = \gamma\Omega_e/\rho$):

$$\begin{aligned} & 2\pi\int_0^\infty d\omega\int_0^\pi d\theta\sin\theta\times\frac{\sqrt{\pi}}{4\sqrt{2}}\;\frac{(Ze)^2\;\Omega_e^2\;qL}{\rho\;c\;\beta_e}\;\times \\ & \times\sum_{n=1}^\infty n|\cos\theta|\left(\frac{J_n}{\sin\theta}-J'_n\right)^2\;\boldsymbol{e}^{-\left(\frac{\omega-|\Omega_e|}{\sqrt{2}\omega\beta_e\cos\theta}\right)^2} = \\ & = \frac{\sqrt{\pi}}{2\sqrt{2}}\frac{1}{3\pi^2}\left(\frac{3}{2}\right)^4\frac{\Omega_e}{c}\;\frac{qL}{\beta_e}\;W_0\;y_0\;\gamma\;\times \\ & \times\int_0^\infty y dy\int_0^{+\gamma}d\psi\psi\;\times \\ & \times\left[\sqrt{1+\psi^2}K_{1/3}(\eta)\;-\frac{1+\psi^2}{\gamma}K_{2/3}(\eta)\right]^2\boldsymbol{e}^{-\boldsymbol{z}^2}\approx \\ & \approx 0.3\;\frac{\Omega_e}{c}\;q\;L\;W_0\;y_0^{4/3}\;, \end{aligned} \quad (35)$$

where

$$\begin{aligned} \boldsymbol{z} =& \frac{n\;\Omega-\Omega_e}{\sqrt{2}\;n\;\Omega\;\beta_e\;\cos\theta} = \frac{y-y_0}{\sqrt{2}\;y\beta_e\psi/\gamma} = \\ =& \frac{1}{\psi\Delta}\left(1-\frac{y_0}{y}\right), \\ \Delta \equiv& \sqrt{2}\beta_e/\gamma,\quad y_0 = \frac{2\Omega_e}{3\Omega\gamma^3}. \end{aligned} \quad (36)$$

Thus we have neglected the term with $K_{2/3}$, because of the factor $1/\gamma$ and have only estimated the term $K_{1/3}$ (π mode) in the following way (confirmed with direct numerical integration for γ up to 300):

$$\begin{aligned} & \int_0^\infty y dy\int_0^{+\gamma}d\psi\psi(1+\psi^2)K_{1/3}^2(\eta)\,\boldsymbol{e}^{-\boldsymbol{z}^2}\sim \\ & \sim y_0^2\Delta\int_0^{+\gamma}d\psi\psi^2(1+\psi^2)K_{1/3}^2(\eta_0)\approx \\ & \approx\frac{\pi}{\sqrt{3}\;2^{1/3}}\;\Gamma(2/3)\;\Delta\;y_0^{1/3} \end{aligned} \quad (37)$$

(here $\eta_0 = \frac{y_0}{2}(1+\psi^2)^{3/2}$).
We have used the following integral:

$$\begin{aligned} & \int_{-\infty}^{+\infty}d\psi\psi^2(1+\psi^2)K_{1/3}^2(\eta_0) = \\ & = \frac{\pi}{\sqrt{3}\;y_0}\left[\int_{y_0}^\infty K_{5/3}(x)dx+K_{2/3}(y_0)\right]\approx \\ & \approx\frac{\pi}{\sqrt{3}\;y_0}\left[\frac{3}{2}\;2^{2/3}\;\Gamma(5/3)\;y_0^{-2/3}-2^{-1/3}\;\Gamma(2/3)\;y_0^{-2/3}\right] = \\ & = \frac{\pi}{\sqrt{3}\;2^{1/3}}\;\Gamma(2/3)\;y_0^{-5/3} \end{aligned} \quad (38)$$

[2] meaning that $j_{\pi,n}$ and $j_{\sigma,n}$ have the same sign, so the vector $\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = Re\;\vec{j}_\omega e^{i\omega t}$ rotates counterclockwise

$$K_\nu(y_0) \approx 2^{\nu-1}\Gamma(\nu)\, y_0^{-\nu} \quad (y_0 << 1).$$

According to (35), the fraction of deposited energy is

$$\frac{\Delta W}{W_0} \approx 0.3\, \frac{\Omega_e}{c}\, q\, L\, y_0^{4/3} \approx \frac{L}{\lambda}\, q\, y_0^{4/3}.$$

This expression scales with the beam energy as $\gamma^{-8/3}$. If the propagation length within the cloud L is fixed, it is inversely proportional to the magnetic field B_0. If L varies according to $L \sim \rho/\gamma$, then the dependence is stronger: $\sim B_0^{-2}$.

LHC parameters

The values of the density parameter q in Table 1 correspond to pessimistic (large) electron density $N_0 = 10^6\ [cm^{-3}]$ (q scales as N_0) and the propagation length within the cloud is taken to be $L \approx 10\ m$. For LHC circulating beam current $\sim 0.5\ A$, the total radiated power is $W_0 \approx 0.06$ W at injection and 3.6 kW at collision, hence the absolute deposited power per beam is negligible.

Table 1: Fraction of deposited energy with LHC parameters.

	p^+ collision	p^+ injection
γ	7460	480
B_0	83860	5390
$\omega = \Omega_e$	$1.5\ 10^{12}$	$9.5\ 10^{10}$
$\lambda[cm]$	0.1	2
Ω	$1.08\ 10^6$	$1.08\ 10^6$
y_0	$2\ 10^{-5}$	$5\ 10^{-3}$
$q\ (N_0 = 10^6\ [cm^{-3}])$	1.510^{-9}	3.510^{-7}
$\Delta W/W_0\ (L = 10\ m)$	$\sim 10^{-12}$	$\sim 10^{-8}$

4 SUMMARY AND CONCLUSIONS

An expression (corrected Schott formula) has been derived for the synchrotron radiation spectrum produced by a relativistic particle, which traverses a large (w.r.t. the wavelength) volume of magnetized plasma (electron cloud in accelerator bending magnet). We have estimated the fraction of absorbed power at frequencies near the first cyclotron resonance due to the presence of resonance electrons (Cherenkov resonance). We found that:

– the absorption would have been stronger in case of an electron traversing an electron cloud, since in such case the stronger σ mode of linear-polarization components of spontaneous radiation decays as (couples with) the extraordinary wave;

– for the realistic case of positively charged beam particle, the absorption occurs away from the median plane and is caused by coupling between the π mode and the extraordinary wave;

– for the case of LHC, both the absorbed power and the effect on the radiated spectrum are negligible.

Our estimations are based on a collisionless plasma model for the cloud, typical (LHC) density and temperature parameters, and Maxwellian velocity distribution of the electrons.

Acknowledgments: D. Kaltchev is grateful to the AP group at CERN for its hospitality, to F. Ruggiero for the support of this work, and to his former supervisor E. Perelstein.

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